SHORT TERM SACRIFICES FOR LONG TERM BENEFITS: A LOOK AT HIGH GROWTH FIRMS

Moren Levesque  
University of Waterloo, Canada, levesque@uwaterloo.ca

Maria Minniti  
Babson College, USA

Recommended Citation
Available at: http://digitalknowledge.babson.edu/fer/vol27/iss10/2
Emerging countries tend to be characterized by large quantities of labor and by rapidly expanding markets. High growth firms typically need to increase their workforce to maintain rapid sales growth. Thus, emerging countries tend to be well suited for high growth firms with labor intensive strategies. Difficult decisions, however, must be made to achieve and sustain high growth and profitability. These decisions involve various tradeoffs, including maximizing hiring in order to generate early profit versus making short term sacrifices and investing in complementary resources in order to generate long term profits. We develop a multi-period decision model of hiring policies for high growth firms that offers new insights on the relationships between profitability, size, and relative size increase in rapidly expanding markets.

INTRODUCTION

Only a small number of firms achieve high growth, and even fewer manage to maintain it over multiple years. In fact, periods of rapid growth are usually short-lived and the strains associated with managing a fast growing firm often produce performance problems (Levesque and MacCrimmon 1999). Although few in number, high growth firms nonetheless contribute very significantly to wealth accumulation (Nicholls-Nixon 2005) and, more importantly, play a very important role in job creation (Birch et al. 1997, Delmar et al. 2003, Storey 1994). The influence and characteristics of high growth firms have been widely studied in developed countries. However, largely because of the lack of data, their relative importance in emerging countries is less understood. Within this context, the relationship between hiring practices and sales revenues is also relatively unknown. We argue that high growth entrepreneurial firms are responsible for a significant portion of economic growth in emerging countries and that, in those countries, because of the availability of relatively less expensive labor, they are more likely to flourish.

In recent years, the rate of economic growth in emerging countries has exceeded that of the rest of the world. Countries such as China, for example, have shown that significant rates of growth are possible even with virtually no R&D expenditure (Hsiao and Shen 2003, Mah 2005). Also, comparing 2002 to 1990, the business environment in China has been found to be more conducive to entrepreneurial activities (Tan 2005). This suggests that an increase in entrepreneurial attitudes, generated by increased incentives, has contributed significantly to economic growth in China. Brazil, India, and a few former Soviet economies have shown similar patterns. Overall, entrepreneurship seem to play a very important role in emerging countries and high growth firms able to produce increasing quantities of output given existing technologies may be responsible for a significant portion of the rapid growth of these economies.

Emerging countries tend to be characterized by large quantities of available labor and by rapidly expanding markets (whether locally or globally). High growth firms typically need to increase their workforce to maintain rapid sales growth. Thus, emerging countries tend to be well suited for labor intensive strategies. Nevertheless, difficult decisions must be made to achieve and sustain high growth and profitability. These decisions involve various tradeoffs, including maximizing hiring in order to generate early profit versus making short term sacrifices and investing in complementary resources in order to generate long term profits. We build a framework that investigates hiring decisions over time and the resulting patterns of firms’ growth in the context of rapidly expanding markets which, although possible in any economy, are more likely in emerging countries. Specifically, we ask how a firm’s earlier
years and current hiring decisions influence its sales revenue’s growth and whether hiring in early years might limit the future growth potential of the firm.

Using a dynamic decision model of hiring policies for high growth firms, we offer some insights on the relationships between profitability, size (in sales revenue and number of employees) and relative size increase. We tailor this decision model to high growth firms (typically smaller and younger firms) by making assumptions on (1) the length of the planning horizon, (2) the change in the (absolute) rate at which sales revenue increases from hiring an additional employee, (3) the objective function of the firm, and (4) the allowable set of hiring decisions. We utilize a decision-theoretic framework to identify conditions under which a high growth firm operating in a rapidly expanding market should increase its hiring. These conditions include an anticipation of higher future profit margins and of a higher firm valuation (e.g. through a buyout).

Our analysis suggests that the cumulative effects of increased hiring in early years, which also results in higher sales revenues in early years, limit the relative growth potential of the firm, where relative growth is computed as the increase in sales revenue from year \( t \) to year \( t+1 \), divided by sales revenue in year \( t \). In other words, the larger a firm becomes, the harder it is to increase its relative growth in sales revenue. The rationale behind this argument is that too little growth may mean that insufficient revenues are coming in to cover the costs of running the business. Whereas too much growth may cause running into financial constraints to pay suppliers, loose key employees, and/or loose control over operations.

THEORETICAL BACKGROUND

Emerging countries are characterized by rapidly expanding domestic and international markets. A rapidly growing market is defined as a market able to absorb an increasing quantity of output without necessarily experiencing a decline in price. These markets are responsible for a large portion of the economic growth of emerging economies. We are interested in the firms that operate in those markets. New firms can be categorized according to their growth ambition. Among them, rapidly growing firms are those characterized by five strategies: (1) product proliferation, (2) mass market proliferation, (3) increasing value to select customers, (4) distribution innovation, and (5) acquisition and consolidation (Mascarenhas et al. 2002). Although other applications are possible (Moreno and Casillas 2007), we focus on high growth firms that, operating in rapidly expanding markets, plan on product proliferation. These firms expect to increase significantly their workforce, especially in their early years of operation (for an updated empirical review of high growth entrepreneurship across countries see Autio 2007). To summarize, we study new firms that face an infinitely elastic demand curve and, as a result, are able to place any quantity of additional output at current prices.

Although we deal with the relationship between firm size and firm growth, models technically similar to ours have been developed in the ramp up literature. Production ramp up is the period of time during which a manufacturing process is scaled up from a small laboratory-like environment to high-volume production. When a company launches a product, the temptation is to immediately ramp up sales force capacity in order to gain market shares as quickly as possible. But hiring a full sales force too early may cause the firm to burn through cash and fail to meet revenue expectations. This is the case because before a firm can sell an innovative product efficiently, it needs to learn how it will be acquired and used (Terwiesch and Wu 2004). Also, the sales learning curve, that is the process describing how employees transfer knowledge and experience back and forth between the production line and purchasing, manufacturing, engineering, planning, and operations, absorb time and may be unwisely accelerated (Leslie and Holloway, 2006). These issues show how the attempt to increase output production in a short period of time may have a negative effect on long term growth.

The issue of ramping up production has been also analyzed in the context of highly seasonal workloads in which firms have to significantly ramp up and down their resources between off season and busy season (Ramanujan and Fisher 2006). In general, shorter product life cycles, more rapid product
obsolescence, and the increasing intensity of global competition have driven firms to strive for a more rapid introduction of new products to market. Many firms, especially in high-tech industries, are shifting their focus from minimizing time-to-market to minimizing time-to-volume (Terwiesch et al. 2001). This puts the tail end of product development, the production ramp up, in a critical position. Carrillo and Franz (2006) have developed a model of several key new product development decisions. In their work, they examine investment strategies related to the timing and duration for investments in both design and process capacity over a given planning horizon. Their model offers guidance regarding the optimal time-to-market and ramp-up time necessary to meet peak demand for the new product. Although the application is different, our model complements the insight provided by these works by focusing in details on the relationship between workforce, rapid increases in production, and characteristics of the market.

Also, although some work on wealth threshold relative to workforce size exists (e.g. Shepp and Shiryaev 1996), the literature on high growth firms lacks models on the development process of high growth startups (Artmann et al. 2001). Thus, our paper contributes to the literature on the relationship between firm size and firm growth (Lotti & Santarelli, 2001). Gibrat’s law states that firm growth is independent of firm size (Gibrat 1931). Our framework proposes a departure from this law by showing that increased hiring in early years increases expected sales revenues, but has a diminishing effect on the relative growth of those revenues. Our model proposes a time-based explanation to the failure of Gibrat’s law, complementing other studies that have also observed failure of this law over the years (e.g. empirical work in the steel, petroleum and rubber tire industries (Mansfield 1962), in the manufacturing sector (Hall 1987, Evans 1987), for young firms in West Germany (Almus and Nerlinger 2002), as well as decision-theoretic works (Gifford 1992a,b, Moreno and Casillas 2007, Segal and Spivak 1989).

Markman and Gartner (2002) found no correlation between high growth and firm profitability but a significant and negative correlation between firm age and profitability, with younger firms experiencing slightly higher profitability rates. Their results are due to the fact that they did not consider the effect of time. Supporting empirical evidence provided by Lévesque and MacCrimmon (1999), our model shows that time play a crucial role in explaining hiring practices and must be considered in order to get an accurate description of the relationship between high growth and firm profitability.

Although preliminary, our study has important implications not only for rapidly expanding markets in general and for those markets in emerging countries in particular. The recent economic growth in countries such as Brazil, China, and India is largely based on the ability exhibited by dynamic entrepreneurs to exploit fast growing markets. In turn, this ability rests, in many cases, on the ability of these entrepreneurs to increase their output and satisfy a large demand by increasing their workforce. Within this context, policy-makers may find attractive the prescriptive aspect of the model when new industries associated with high velocity development – and thus rapid generation of new jobs – can be identified.

A MODEL OF LONGITUDINAL GROWTH

We present a model in which a high growth firm must decide over time on workforce size in order to maximize its expected valuation (a function of expected profits and of sales revenue expected at the end of the planning horizon). Since the firm’s workforce decisions highly influence its production outputs, outputs represents the state of this decision system. However, in a rapidly expanding market, given the assumption of an infinitely elastic demand curve and constant prices, setting total outputs is equivalent to setting sales revenues. Consequently, and consistently with existing literature (Lévesque and MacCrimmon, 1999), the state of the decision system can be interpreted as sales revenues. We formulate the decision problem as a discrete-time, discrete-state Markov process, and, for each period of time, we identify the hiring policy that describe for each level of sales revenue the best hiring decision.
The decision framework we present in this paper was originally developed by Lévesque and MacCrimmon (1999) and, where applicable, we preserve the notation they used. However, our framework differs from their in several important ways. First, we specify the dynamics of sales revenues over time, whereby the value of the marginal product of labor (affecting those dynamics) is generalized and not limited to be dependent on a firm’s profit margin. Second, we account for the value of sales revenue in the firm’s valuation to be discounted. Third, in our framework, profit margins are no longer random since in rapidly expanding markets, competition does not force firms to reduce price, thus reducing uncertainty. Finally, while Lévesque and MacCrimmon concluded their contribution by testing their decision model on a sample of 51 high growth firms in the United States, we articulate a theoretical rationale for the optimal hiring policy and its qualitative prescriptions and their implications for rapidly expanding markets.

**Notation, Sales Dynamics and Expected Firm Valuation**

A firm’s hiring decision is made at each time period \( t \) (e.g. a month, a quarter, or a year) over a finite planning horizon of length \( T \). The status, or state, of the decision system used to guide the hiring decision is measured by a period of sales revenue and denoted by \( r \). The state space, denoted by \( \mathbb{R} \), is a discrete set of all positive real numbers that are possible values for \( r \). Given state \( r \), in this decision system an allowable action represents one period of hiring and is denoted by \( \Delta w \). The set of allowable actions, denoted by \( W(r) \), is a set of non-negative integers that are possible values for \( \Delta w \) (the set of allowable actions will be further characterized and made time dependent). A (randomized) decision rule, denoted \( d_t \), is a mapping of the state space \( \mathbb{R} \) into the set of probability distributions on the action space \( W \). A hiring policy \( \phi = \{ d_t : t = 1, 2, 3, \ldots, T \} \) prescribes for each period \( t \) a decision rule \( d_t(r) = \Delta w \in W(r) \).

The (absolute) rate at which sales revenue increases over time, \( r_{t+1} - r_t \), depends on the hiring decision: the more a firm hires in the current period, the higher \( r_{t+1} - r_t \) (due to an increased amount of produced outputs). However, the change in the rate at which sales revenue increases from hiring an additional employee, denoted by \( Z_t \) and henceforth referred to as the value of the marginal product of labor, may depend on factors such as employees’ performance and effort that may not be under the complete control of the firm. Thus, \( Z_t \) is a non-negative random variable, and the hiring decision has a randomized effect on the growth of sales revenue (in standard economic modeling, this issue would be framed as a principal-agent problem but such a formulation would not be suitable for our purpose). In addition, on any given time period, other factors, such as technological innovations, may increase or decrease the change in sales revenues. As a result, we also include in to the sales dynamic equation a random noise factor denoted by \( X_t \), with a mean effect of zero. Given state \( r_t \in \mathbb{R} \) for period \( t \) and a non-randomized decision rule \( d_t(r) = \Delta w \), sales revenue at \( t+1 \), \( r_{t+1} \), satisfies

\[
r_{t+1} - r_t = Z_t \Delta w + X_t.
\]

The dynamic characterizing (1) is Markovian as the state of the decision system at \( r_{t+1} \) depends on the past through the current period \( t \) only.

Lévesque and MacCrimmon (1999) found some support for similar sales revenue dynamics from operationalizing \( Z_t \) as an inversely proportional function of 1 minus the profit margin, and \( X_t \) as the increment in that function over time. This operationalization is restrictive, however, as it forces profit margin to be random.

The firm’s objective is to maximize its expected valuation expressed as the sum of expected total discounted profit at the end of the planning horizon and a salvage value that accounts for the level of sales revenue expected to be reached by the end of the planning horizon. Specifically, the firm’s immediate
profit is \( R(t, d_t(r_t)) = m_t r_t \), where \( m_t \) is the profit margin in period \( t \). Thus, the firm’s valuation over the decision-making horizon if hiring policy \( \varphi \) is used and \( r \) is the initial sales revenue is

\[
\nu^\varphi_t(r) = E^\varphi \left[ \sum_{t=1}^{T} \mathcal{Z} R_t(r_t, d_t(r_t)) + \mathcal{Z}^{T+1} R_{T+1}(r_{T+1}) \mid r_0 = r \right],
\]

where \( E(\cdot) \) is the expectation operator and \( \lambda \) a discount factor. As many high growth firms report selling out to another firm as their exit strategy, the salvage value can be used to reflect an anticipated valuation of the firm for buyout. We thus define a non-negative real number \( I \) as the buyout index where

\[
R_{T+1}(r_{T+1}) = I \cdot r_{T+1}
\]

for any \( r_{T+1} \in \mathbb{R} \). Hence, \( \nu^\varphi_t(r) \) represents the present value of the profit obtained from sales of the firm’s products or services (if \( I = 0 \)), and/or the market capitalization of the firm at the time of a buyout. We then characterize a hiring policy \( \varphi^* \) for which

\[
\nu^\varphi_t(r) \geq \nu^\varphi_t(r) \quad \text{for any } r \in \mathbb{R} \text{ and for all hiring policies } \varphi; \text{ in other words, the optimal hiring policy.}
\]

**Optimality Equations, Hiring Thresholds and the Optimal Policy**

We derive a hiring policy that identifies the hiring decision over time as a function of sales revenue. The set of values for the marginal product of labor, \( \{Z_t\}_{t=1,2,...,T} \), is a set of \( T \) discrete random variables (due to e.g. employees’ performance), each defined over a set denoted by \( \mathbb{N} \). Similarly, the set of noises, \( \{X_t\}_{t=1,2,...,T} \), is composed of \( T \) discrete random variables (due to e.g. technological changes), each defined over a set denoted by \( \Xi \). If \( f(Z;X) \) represents the joint probability function of that value and noise at \( t \), then the probability that sales revenue is \( j \) in period \( t+1 \) while \( \Delta w \) employees are hired when sales revenue was \( r \) in period \( t \) is

\[
p_t(j \mid r, \Delta w) = P((Z_t, X_t) \in \{(z, x) \in \mathbb{N} \times \Xi : j = r + z \Delta w + x\}) = \sum_{\{(z, x) \in \mathbb{N} \times \Xi : j = r + z \Delta w + x\}} f_t(z, x).
\]

\( p_t(j \mid r, \Delta w) \) is the transition probability function of the Markov decision process.

We utilize the backward induction method to solve this decision model (for more details on backward induction and Markov decision processes in general see Puterman (1994)). Let \( v^*_t(r_t) \) be the expected total discounted profit from \( t \) onward if the optimal hiring policy is used from \( t \) onward and the sales revenue is \( r_t \) at \( t \). We have,

\[
v^*_t(r_t) = \operatorname{Max}_{\Delta w \in W(r_t)} \left\{ R_t(r_t, \Delta w) + \sum_{j \in \mathbb{R}} \lambda p_t(j \mid r_t, \Delta w) \cdot v^*_t(r_{t+1}) \right\}, \quad t = 1, 2, ..., T,
\]

with \( v^*_t(r_{T+1}) = R_{T+1}(r_{T+1}) \) since no more decisions are taken after the last period. Using (3) and \( R_t(r_t \Delta w) = m_t \cdot r_t \), it is possible to verify that (4) is equivalent to

\[
v^*_t(r_t) = \operatorname{Max}_{\Delta w \in W(r_t)} \left\{ m_t \cdot r_t + \sum_{(z, x) \in \Xi \times \mathbb{N}} \lambda f_t(z, x) \cdot v^*_t(r_{t+1} + z \Delta w + x) \right\}, \quad t = 1, 2, ..., T
\]

and

\[
v^*_t(r_{T+1}) = I_{T+1} \quad \text{for any } r_{T+1} \in \mathbb{R}.
\]

Equations (5) and (6) describe the optimality equation system that is solved next.
Given an infinitely elastic demand curve, price and wages are assumed constant. In a rapidly expanding market, especially if located in an emerging country, this means that labor becomes more attractive than other production inputs. As a result, a firm has an incentive to hire as much as possible in early years to increase its sales revenue. In the long run, however, that early decision may limit the firm’s ability to further increase its workforce because of its insufficient early investment in non-labor production inputs. In addition, in rapidly growing markets, the infrastructure (utilities, means of transportation, financial services, etc.) may also constrain a firm’s ability to hire. As a result, we define an upper bound on the number of employees a firm can hire in each period. However, over time, as a country’s wealth increases and/or the industry matures, non-labor production inputs may become more available in terms of both quantity and price. Thus, following empirical evidence presented by Lévesque and MacCrimmon (1999), we assume that upper bound to increase exponentially as the finite planning horizon unfolds. For \( t = 1, 2, \ldots, T \), \( W(r_t) = \{ \Delta w_t \in Z^+ \cup \{0\} : 0 \leq \Delta w_t \leq \theta e^{\kappa t} \} \) represents the set of possible actions in period \( t \) given sales revenue \( r_t \). The parameter \( \theta \) is an initial \((t = 0)\) hiring upper bound, whereas \( \kappa \) is the rate at which that bound increases over time. For any sales revenue \( r \in \mathbb{R} \), the optimal hiring policy \( \varphi^* = \{ d_t^* : t=1,2,3,\ldots T \} \) is

\[
d_t^*(r) = \Delta w_t^* = \theta e^{\kappa t} \text{ if } \frac{\sum_{i=t+1}^{T} \lambda (i-t-1) m_i + \lambda T-I > 0}{} \text{ when } 1 \leq t < T, \text{ or if } t = T ; \ d_t^*(r) = 0 \text{ otherwise.} \tag{7}
\]

All technical details are presented in the Appendix.

The optimal policy suggests that hiring occurs at its upper bound level as long as the firm faces a prosperous future, which makes good intuitive sense. The policy also shows that a prosperous future occurs when the discounted sum of the firm’s future profit margins \( (m_i, i = t+1, t+2, \ldots, T) \) and buyout index \( (I) \) is positive. In other words, if the index is 0 (as some firms may not value a buyout exit) then the firm cannot afford all profit margins to be negative in the future. We also note that when these margins are negative and the index small enough, the solution is zero growth (no hiring) but shifts to its maximum as the firm gets close enough to \( T \) for \( I \) to dominate. This illustrates a boundary condition of the model where all firms are worth something if \( I > 0 \) and have a last chance toward the end of the planning horizon to hire and take advantage of it via a buyout.

The optimal hiring policy is thus constrained by future profitability, but does not directly depend on the state of the system, namely sales revenue \( r \). The solution of the model – either no hiring or the assumed maximum possible hiring – illustrates the fact that the selected objective function from period \( t \) onward is linear in the hiring decision at period \( t \) (the optimality equations 5 and 6 are shown in the appendix to be linear in the hiring decision). This could be viewed as a limit of the model, as corner optimal solutions prevent us from observing additional potential (time-independent) tradeoffs associated with hiring decisions and their immediate effects on profitability and firm growth. However, one of our main contributions rests on the consideration of time and the crucial role time plays in explaining hiring practices in order to get an accurate description of the relationship between profitability and growth. Therefore, although limiting, the lack of an interior optimal solution enables us to easily single out time-dependent tradeoffs which would be more difficult to isolate otherwise.

**HIRING AND GROWTH IMPLICATIONS**

The investigation of how changes in key model parameters affect the optimal hiring policy, as well as various growth patterns from implementing that policy, provides additional insights on the relationship between firms’ profitability, size, and relative size increase in rapidly expanding markets. We first note that the profitability condition in (7) is easier to satisfy whenever the profit margins \( m_i \), for \( i = t+1, t+2, \ldots, T \), increase, or whenever the buyout index \( I \) increases. These relationships are established in Proposition 1.
Proposition 1: If implementing the optimal hiring policy, 

(a) a firm has an incentive to increase its workforce when future profit margins are expected to be higher, 
(b) and increases in the buyout index are expected to result in more hiring for any year.

Proposition 1 part (a) is rather intuitive and states that if a firm expects high future returns, it has strong incentives to maximize hiring in order to take advantages of possible high profits. Part (b), instead, states that a highly valued buyout index can overcome multiple successive years of negative profit margins. This may provide an explanation for dot.com companies that, in spite of being unprofitable for multiple years, grew their workforce (and hence sales revenue) rapidly during that same period. High growth firms with multiple years of negative profit margins, yet large workforce sizes, have also been observed in the Lévesque and MacCrimmon’s (1999) sample. For example, between 1990 and 1997 MedicaLogic, which developed and distributed electronic-medical-record software, reported a negative profit margin for every single year, yet grew its workforce from 2 to 147 employees.

By next focusing on firms that expect to experience non-negative profit margins and stable values for their marginal product of labor (say \( E[Z_t] = C \) for any \( t \)), we are also able to characterize the patterns of sales and workforce growth when the optimal hiring policy is implemented. We first compute the relative growth in sales revenue as the increase in sales revenue from year \( t \) to year \( t+1 \), divided by sales revenue in year \( t \). Then, from the sales dynamic equation (1) and from implementing the optimal hiring policy, it is straightforward to verify that the optimal relative growth in sales revenue decreases over time if and only if \( r_0 < C \theta \left( e^x - 1 \right)^2 \), making this growth path monotone over time. When the condition on \( r_0 \) is satisfied, the optimal relative growth is expected to decrease as the firm ages. Similarly, we show that the optimal relative growth in workforce decreases over time if and only if \( w_0 < \theta \left( e^x - 1 \right)^2 \). Hence, the path of workforce growth is expected to be monotone over time and, when the condition on \( w_0 \) is satisfied, the optimal relative growth is expected to decrease as the firm ages. Also, the optimal relative growth in sales revenue can be expressed as a function of the optimal workforce size \( w_t^* = w_0 + \sum_{i=0}^{t-1} \Delta w_t^* \) and shown to be a decreasing convex function of that size. These relationships are established in Proposition 2.

Proposition 2: If profit margins are non-negative and the expected values of the marginal product of labor is constant, when implementing the optimal hiring policy 

(a) the relative growth in sales revenue and in workforce is expected to be monotone over time, and decreasing for firms with small initial sales revenues and small initial workforces, 
(b) and the relative growth in sales revenue is expected to decrease (at a diminishing rate) as the firm’s workforce size is increased.

At the beginning of their life, most firms (whether or not they are high growth firms) experience negligible sales revenue and low workforce levels. Thus, the conditions on \( r_0 \) and \( w_0 \) described in part (a) are most likely to be satisfied and the relative growth in sales revenue, or in the size of the workforce, is expected to decrease as the firm ages. These monotone decreasing relationships are puzzling, as one would expect, instead, an inverted U-shaped relationship, whereby the relative growth increases over time until it reaches a peak and then start decreasing. Our analysis suggests that the cumulative effects of increased hiring in early years, which also results in higher sales revenues in early years, limit the relative growth potential of the firm. This is the case because of the tradeoff between labor and other production inputs and infrastructure described earlier. The negative relationship between growth and age is also consistent with findings by Evans (1987) and Gifford (1992b).
In part (b), a firm’s relative growth in sales revenues is expected to decrease with the number of its employees. Our analysis suggests again that the cumulative effects of increased hiring in earlier years limit the relative growth potential of the firm. This negative relationship is also supported by Evans (1987). Perhaps high-growth firms realize they can increase sales revenue without incurring the higher costs generated by increasing their workforce. This difference could be, again, the result of substituting other production inputs for labor.

**DISCUSSION**

This time-based model provides some suggestions on the tradeoff faced by firms between high growth and profitability. These decisions involve various tradeoffs, including maximizing hiring in order to generate early profit versus making short term sacrifices and investing in complementary resources in order to generate long term profits. We derive an optimal hiring policy. The sales revenue dynamics suggested in Equation (1) imply that the effect of hiring in early years on current expected sales revenue is positive while that on current expected relative growth in sales revenue is negative. Analogous implications are suggested for the effect of the value of the marginal product of labor on current expected sales revenues and growth. In rapidly expanding markets, a high value of the marginal product of labor in early years encourages firms to expand labor at the expenses of other factors of production. This is particularly true in emerging economies where labor tends to be abundant and have lower initial costs than other resources. As a result, firms may find themselves weakened over time with respect to resources other than labor and may find it more difficult to achieve further increases in sales revenues. Thus, high growth firms in rapidly expanding markets face an important tradeoff when deciding how fast to increase their workforce.

Second, our model highlights the fact that high growth firms may have strong incentives to increase their workforce even when generating negative profit margins. The profitability condition expressed by Inequality (7) sheds some light on why high growth firms may choose maximum increase for the size of their workforce, even though they may be unprofitable over multiple years. In fact, a firm’s valuation depends on accumulated profits and on the level of sales revenue at the end of the planning horizon. Since sales revenues are expected to increase over time, the highest level of sales is expected to be at the end of the planning horizon. When multiplied by the buyout index, that expected level determines the portion of the firm’s value that comes from its sales revenue level (as opposed to its accumulated profits). Thus, the profitability condition highlights the fact that the losses in a firm’s value from multiple years of negative profit margins can potentially be compensated by that firm’s buyout index. In other words, the level of sales revenues reached by the firm may be more important for a firm’s valuation than its accumulated profits.

Third, we derive a time-based explanation for the relationship between firm size (in terms of both sales revenue and size of the workforce) and firm growth (in terms of relative sales revenue). When profit margins are non-negative and the expected values of the marginal product of labor is constant, firms with small initial sales revenue and small initial workforce (which is typical of start-ups) will experience decreasing positive growth patterns (in both sales revenue and number of employees) as they age. This is the case because the cumulative effects of increased hiring in early years, which also results in higher sales revenues during those years, limit the relative growth potential of the firm over time. For that same reason, these firms are also expected to experience a decrease in their relative growth in sales revenues as the size of their workforce increases.

Our model is a first attempt to study the relationship between growth decisions and hiring dynamics and, far from closing the issue, it lends itself to several extension and improvements. First, the framework presented can be extended to include explicitly growth determinants other than labor. Physical, financial, and social capital (Hambrick and Crozier 1985), the state of the market growth (Siegel et al. 1993), and the industry cycles should all be considered to better capture the complexity of firm growth decisions.
Second, we have assumed the decision strategy of the high growth firm to be independent from the decision strategies of competing firms. Yet, workforce growth decisions of competing firms may significantly affect that of the firm, as the cost and availability of labor is likely to shift as competing firms simultaneously make hiring decisions. Thus a further extension of our model could include the introduction of a competitive element. Finally, biases may exist in our measurement of growth. The growth implications of our model are somewhat biased in favor of firms with low sales in early years. Although biases also exist for sales per employee (e.g. learning takes time), such a formulation should be explored as it may provide a more accurate measure of firm growth.

The goal of this preliminary research was to investigate short term sacrifices for long term benefits in high growth firms. To achieve this goal, we developed a dynamic decision-theoretic model of hiring policies for these firms that attempted to offer new insights on the relationships between profitability, size and its relative growth. We believe this work provides a small but important benefit to new entrepreneurs as they can better tailor their hiring decisions over time for continuous high growth, to financiers as they can better foresee the benefits of financing the growth of a firm, and to policy-makers as they may better adjust their policies to the rapid generation of new jobs.

Although the relationship between entrepreneurial activity and economic growth is often given for granted, the exact nature of such a relationship and the channels that allow entrepreneurial activity to influence growth are still unknown. In this paper, we consider the importance of high growth firms in a type of markets often observed in emerging economies. Namely, one characterized by an infinitely elastic demand. Our results suggest that the presence of high growth entrepreneurs is a very important condition for economic growth since those entrepreneurs are the ones who mobilize labor. Of course, the supply of inexpensive labor per se is neither sufficient nor necessary to explain the Chinese miracle. What matters is the fact that this labor is mobilized to generate significant increases in output and that the rapidly increasing nature of the global market allows high growth companies to place their product without facing declining prices. It is the presence of entrepreneurs that, by mobilizing this abundant resource, creates growth. This is further supported by the fact that many countries with an abundant supply of low wage labor (such as many African economies) are stuck in low economic growth traps.

The findings of this research will be of interest to policy makers, providers of capital and entrepreneurs. Each of these stakeholders has an interest in firm growth and survival. The more we know about the determinants of each of these phenomena, the greater the opportunities to optimize firm growth while avoiding firm exit. Greater knowledge about the dynamics of growth, survival and exit may enable entrepreneurs to make better strategic decisions, particularly in the hazardous early years. And, if growing businesses are exiting due to capital constraints, this will surely be of interest to policy makers interested in fostering economic growth.

CONTACT: Moren Lévesque; levesque@uwaterloo.ca; (T): 519 888 4567 ext 35367; University of Waterloo, Waterloo, Ontario, Canada N2L 3G1.

REFERENCES


APPENDIX

From applying the backward induction method to Equations (4) and (5), we obtain

\[ v_{t+1}^*(r_{T-t}) = I \cdot r_{T-t} \quad \text{and} \quad v_t^*(r_t) = \max_{\Delta w \in \mathbb{N}^x T(t)} \left\{ m_t r_t + \sum_{(z,x) \in \mathbb{N}^x} \lambda I \cdot (r_t + z \Delta w + x) \cdot f_t(z,x) \right\}. \]

With \( I > 0 \) and \( f_t(z,x) \geq 0 \ \forall \ (z,x) \), \( \Delta w_t^* = \theta e^{\xi t} \) and \( v_t^*(r_t) = \left[ m_t + \lambda I \right] \cdot r_t + \lambda I \cdot \left[ E(Z_t) \cdot \theta e^{\xi t} + E(X_t) \right] \).

It follows that,

\[ v_{t-1}^*(r_{T-t}) = \max_{\Delta w \in \mathbb{N}^x T(t-1)} \left\{ \sum_{(z,x) \in \mathbb{N}^x} \lambda I \cdot (r_{t-1} + z \Delta w + x) + \lambda I \cdot \left[ E(Z_{t-1}) \cdot \theta e^{\xi t} + E(X_{t-1}) \right] \right\} \cdot f_{t-1}(z,x), \]

i.e., \( v_{t-1}^*(r_{T-t}) = \left\{ m_{t-1} + \lambda m_{t-1} + \lambda^2 I \right\} \cdot r_{t-1} + \lambda m_{t-1} + \lambda I \cdot \left[ E(Z_{t-1}) \cdot \theta e^{\xi t} + E(X_{t-1}) \right] \),

which is linear in \( \Delta w \). Hence, the optimal hiring policy at period \( T-1 \) is \( \Delta w_{T-1}^* \) if \( m_{T-1} + \lambda I > 0 \) and 0 otherwise.

\[ v^*_{t-1}(r_{T-t}) = \begin{cases} \theta e^{\xi (T-t)} & \text{if } m_{T-1} + \lambda I > 0, \\ 0 & \text{otherwise} \end{cases} \]

This provides the optimal reward from period \( T-1 \) onward. Similarly, from (A1),

\[ v_{t-2}^*(r_{T-t}) = \begin{cases} \max_{\Delta w \in \mathbb{N}^x T(t-2)} \left\{ m_{t-2} + \lambda m_{t-2} + \lambda^2 m_{t-1} + \lambda I \right\} \cdot r_{t-2} + \lambda m_{t-1} + \lambda m_{t-1} + \lambda^2 I \cdot \left[ E(Z_{t-2}) \cdot \Delta w + E(X_{t-2}) \right] + \lambda^2 I \cdot \left[ E(Z_{t-2}) \cdot \theta e^{\xi T(t-1)} + E(X_{t-2}) \right] \right\} & \text{if } m_{T-1} + \lambda I > 0, \\ \max_{\Delta w \in \mathbb{N}^x T(t-2)} \left\{ m_{t-2} + \lambda m_{t-2} + \lambda^2 m_{t-1} + \lambda I \right\} \cdot r_{t-2} + \lambda m_{t-1} + \lambda m_{t-1} + \lambda^2 I \cdot \left[ E(Z_{t-2}) \cdot \Delta w + E(X_{t-2}) \right] + \lambda^2 I \cdot \left[ E(Z_{t-2}) \cdot \theta e^{\xi T(t-1)} + E(X_{t-2}) \right] \right\} & \text{otherwise}. \]

Regardless of the sign of \( m_{T-1} + \lambda I \), one can chose \( \Delta w_{t-2}^* \) if \( \sum_{i=0}^1 \lambda^{i+1} m_{t-1} + \lambda I > 0 \) and thus

\[ v_{t-2}^*(r_{T-t}) = \left\{ \sum_{i=0}^{t-1} \lambda^{i+1} m_{t-1} + \lambda I \right\} \cdot r_{t-2} + \lambda I \cdot \left[ E(Z_{T-1-t}) \cdot \theta e^{\xi T(t-1-j)} + E(X_{T-1-t}) \right] \]

\[ \quad + \lambda \sum_{j=0}^{t-1} \left\{ \sum_{i=0}^j \lambda^{i+1} m_{t-1} + \lambda I \right\} \cdot \left[ E(Z_{T-1-j}) \cdot \theta e^{\xi T(t-1-j)} \cdot Y_{\sum_{i=0}^j \lambda^{i+1} m_{t-1} + \lambda I > 0} + E(X_{T-1-j}) \right] \right\}, \]

where \( Y_{\sum_{i=0}^j \lambda^{i+1} m_{t-1} + \lambda I > 0} \) is an indicator variable with \( Y_{\sum_{i=0}^j \lambda^{i+1} m_{t-1} + \lambda I > 0} = \begin{cases} 1 & \text{if } \sum_{i=0}^j \lambda^{i+1} m_{t-1} + \lambda I > 0, \\ 0 & \text{otherwise}. \end{cases} \)
Therefore, the postulated reward from period $t = k$ onward, $k \in \{1,2,\ldots,T\}$. Then, we use this optimal reward to identify the optimal hiring strategy at period $k-1$, and the optimal reward from $k-1$ onward. Specifically, we posit that

$$\Delta w^*_k = \begin{cases} 
\theta e^{\delta k} \text{ if } \sum_{i=0}^{T-k} \lambda^{T-k-i} m_{T-i} + \lambda^{T-k} I > 0 \quad \text{and} \quad v_r^*(r) = \left[ \sum_{i=0}^{T-k} \lambda^{T-k-i} m_{T-i} + \lambda^{T-k} I \right] \cdot r_k + \lambda^{T-k} \left[ E(Z_T) \cdot \theta e^{\delta T} + E(X_T) \right]
\end{cases}$$

$$+ \lambda \sum_{j=0}^{T-k-1} \left[ \sum_{i=0}^{j} \lambda^{j-i} m_{T-i} + \lambda^{j+1} I \right] \cdot \left[ E(Z_{T-j}) \cdot \theta e^{\delta(T-j)} \cdot Y_{\left[ \sum_{i=0}^{j} \lambda^{j-i} m_{T-i} + \lambda^{j+1} I \right]} + E(X_{T-j-1}) \right]. \quad (A2)$$

We show that given (A2) at $t = k$, for $t = k-1$, $\Delta w^*_{k-1} = \begin{cases} 
\theta e^{\delta(k-1)} \text{ if } \sum_{i=0}^{T-k} \lambda^{T-k-i} m_{T-i} + \lambda^{T-k} I > 0 
\end{cases}$

$$0 \text{ otherwise}$$

and

$$v^*_{k-1}(r_{k-1}) = \left[ \sum_{i=0}^{T-k-1} \lambda^{T-k-1-i} m_{T-i} + \lambda^{T-k+1} I \right] \cdot r_{k-1} + \lambda^{T-k+1} \left[ E(Z_{T-j}) \cdot \theta e^{\delta(T-j)} \cdot Y_{\left[ \sum_{i=0}^{j} \lambda^{j-i} m_{T-i} + \lambda^{j+1} I \right]} + E(X_{T-j-1}) \right]. \quad (A3)$$

From (A2),

$$v^*_{k-1}(r_{k-1}) = \max_{\Delta w \in \partial \delta + (k-1)} \left\{ m_{k-1} r_{k-1} + \sum_{(z,x) \in \mathbb{Z}} \lambda \left[ \sum_{i=0}^{T-k} \lambda^{T-k-i} m_{T-i} + \lambda^{T-k+1} I \right] \cdot \left[ r_{k-1} + z \Delta w + x \right] + C \cdot f_{k-1}(z,x) \right\}$$

$$= \max_{\Delta w \in \partial \delta + (k-1)} \left\{ \sum_{i=0}^{T-k} \lambda^{T-k-i} m_{T-i} + \lambda^{T-k+1} I \right\} \cdot r_{k-1} + \lambda \left[ \sum_{i=0}^{T-k} \lambda^{T-k-i} m_{T-i} + \lambda^{T-k+1} I \right] \cdot \left[ E(Z_{k-1}) \cdot \theta e^{\delta k} + E(X_{k-1}) \right] + \lambda C \right\}$$

where $C$ is a constant (independent of the selection for $\Delta w$) equal to

$$\lambda^{T-k+1} \left[ E(Z_T) \cdot \theta e^{\delta T} + E(X_T) \right] + \sum_{j=0}^{T-k-1} \left[ \sum_{i=0}^{j} \lambda^{j-i} m_{T-i} + \lambda^{j+1} I \right] \cdot \left[ E(Z_{T-j-1}) \cdot \theta e^{\delta(T-j)} \cdot Y_{\left[ \sum_{i=0}^{j} \lambda^{j-i} m_{T-i} + \lambda^{j+1} I \right]} + E(X_{T-j-1}) \right].$$

Therefore, the postulated $\Delta w^*_{k-1}$ provides an optimal hiring strategy at $t = k-1$ and

$$v^*_{k-1}(r_{k-1}) = \left[ \sum_{i=0}^{T-k} \lambda^{T-k-i} m_{T-i} + \lambda^{T-k+1} I \right] \cdot r_{k-1} + \lambda \left[ \sum_{i=0}^{T-k} \lambda^{T-k-i} m_{T-i} + \lambda^{T-k+1} I \right] \cdot \left[ E(Z_{k-1}) \cdot \theta e^{\delta(k-1)} \cdot Y_{\left[ \sum_{i=0}^{j} \lambda^{j-i} m_{T-i} + \lambda^{j+1} I \right]} + E(X_{k-1}) \right] + \lambda C,$$

which by regrouping terms is equivalent to (A3).