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Production, Process Investment and the Survival of Debt Financed Startup Firms

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PRODUCTION, PROCESS INVESTMENT AND THE SURVIVAL OF DEBT FINANCED STARTUP FIRMS

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Whether to invest in process development that can reduce the unit cost and thereby raise future profits or to conserve cash and reduce the likelihood of bankruptcy is a key concern faced by many startups firms that have taken on debt. We explore this concern by examining the production quantity and cost-reducing R&D investment decisions in a two period model. A startup firm must make a minimum level of profits at the end of the first period to survive and continue operating in the second period. We show that under a base case, with deterministic demand, such a startup should produce the monopoly quantity and use an *invest-all-or-nothing* investment policy. However, under stochastic demand and allied survival constraint, the optimal investment policy does not necessarily have an *all-or-nothing* character. We establish conditions for creating hedges through either aggressive or conservative investment alternatives. If the startup makes a “conservative” investment decision, it sacrifices some first period expected profits to increase its survival chances and chooses an optimal quantity less than the monopoly quantity. Further, if the startup decides to invest “aggressively,” then it produces more than the monopoly quantity to cover the higher bankruptcy risk due to such aggressive investment.

Keywords: Joint Production and Process Investment Decisions; Operational Hedging; Startup Operations; Survival under Debt.

1. Introduction

According to a study by U.S. Bureau of the Census, it is estimated that over 700,000 startups are formed every year in the US (Acs and Armington 1998). However, only a small proportion of these startups are able to grow their revenues and become profitable, and even a smaller proportion of these firms can show continued growth and make initial public offerings (Acs and Armington 2003). Startup firms are endowed with unique characteristics regarding their asset structure, organization type and growth orientation (Gifford 2005), and their operational decisions are often restricted by debt and other financial considerations (Berger and Udell 2005). In practice, most startups have very limited access to capital. Most of these firms take on debt and face immediate bankruptcy in case of a payback default. Hence, for

startup managers it is necessary to generate adequate short-term cash flows by exploiting immediate business opportunities in order to keep up with the cash outflows and avoid bankruptcy.

Further, startups are not merely focused on survival. They are also interested in long-term growth. Indeed, most startup firms are concerned with their ability to invest in research and development (R&D) to improve their products and services (Bhide 2000). While such investments may not generate immediate cash flows, they are likely to improve the future prospects of the firm. In general, under bankruptcy risk, long-term growth and short-term survival are two intimately linked concerns. A key area of startup decision making, involving short term survival against long term growth, is the R&D investment that is aimed at reducing the firm's unit production cost. In this paper, we investigate the optimal operating decisions of a startup under debt which can invest in production to exploit the current business opportunities and generate short-term cash flows, or, it can also have a strategy under which it may also invest in process improvement to secure future market share and long-term profits.

We have conducted a series of interviews in order to understand the key considerations that affect process R&D investment decisions in startup settings. For example, Faradox is an Austin based startup which provides high energy density capacitors using its niche production process. Faradox views process development to reduce unit cost as a key competitive aspect of its business. During our interviews, the VP of business operations at Faradox stated that there was tremendous amount of on-going research in the field of high energy density capacitors and, it was quite likely that new competitors might enter the market by developing new and possibly more efficient production processes with lower unit costs. He also acknowledged that while process R&D was a key element of long term survival of Faradox, it was very costly and its return was highly uncertain. Further, while making investment decisions, predicting consumer demand also imposes a serious challenge for this company since the market is evolving and the customer base is hard to analyze. Allied issues have also surfaced at other Austin startups, AccuWater, AxsTracker, Big Foot Networks etc. Managers at these firms indicated that their production and investment decisions are affected by risk created by cash flow and technology performance (Erzurumlu

and Tanrisever 2007). These concerns are consistent with descriptions of startup decision making in the extant literature (Bhide 2000, Shane 2007).

However, in the absence of a modeling framework, these managers are not able to assess their production and process improvement risks, and underlying tradeoffs, with precision. This has motivated our effort to formalize a class of factors that have been central for startup companies while choosing their operating policies regarding production and process investment in the presence of survival considerations: *uncertainties surrounding demand, technological performance and likely entry of competition*. These factors form the core of our model, and we examine their impact on the selection of operating (production and process investment) policies and the survival chances of the startup. For ease of exposition, model specification and analysis are developed in two stages. In the first stage, we analyze a base case (BC) regarding our operating decisions under deterministic demand with a two-period model. BC provides benchmarks for more involved models. In the second stage of our analysis, we allow stochastic realization of demand. This is termed as the stochastic demand and survival constraint (SDSC) case. With stochastic demand, profits after the first period are not guaranteed and a probabilistic survival constraint comes into play. SDSC is amenable to closed form solutions under limited conditions. Hence, we explore the underlying tradeoff between expected profit and bankruptcy risk through a combination of analytical and numerical solutions.

The contributions from our work are threefold. First, we specify a deterministic-demand model for a debt financed startup firm as a base case, and characterize an optimal *invest-all-or-nothing* policy which derives the conditions for investment in process improvement in order to enhance long-term profits. Second, with demand uncertainty and the consequent probabilistic survival constraint, we find that such a startup responds to the bankruptcy risk by increasing the investment threshold, i.e., the firm looks for more favorable market conditions to invest. Indeed, while balancing the bankruptcy risk with future growth opportunities, the startup may either behave *conservatively (aggressively)* by investing and producing less (more) than the BC level. In effect, a probabilistic survival constraint induces the startup to produce so as to create an *operational hedge* with respect to its process investment decision. Further, we

offer a probabilistic survival measure that reflects the riskiness of the startup's operating decisions under the threat of bankruptcy. Third, we explore the impact of the existence of process investment opportunities, immediate profitability of the firm and limited debt availability on the optimal operating decisions and the allied survival chances. In addition, we have circled back to some startup managers and sought their feedback on our findings. We discuss the managerial implications of these findings while we synthesize and discuss our results.

The rest of our paper is organized as follows. §2 provides a review of the related literature. In §3 we analyze the BC and characterize a closed form solution under deterministic demand. We extend our discussion to SDSC case in §4. In §5 we discuss limited debt capacity. §6 addresses managerial implications, limitations and concludes our paper.

2. Relevant Literature

Here we briefly review the streams of literature that are closely related to our work: investment in process R&D, startup operations and financing, and the entrepreneurial decision-making.

Investment in process R&D and allied cost reduction and capacity management decisions have long been key issues in the manufacturing technology management literature (De Groote 1988, Fine and Porteus 1989, Chand et al. 1996, Li and Rajagopalan 1998, Carrillo and Gaimon 2000, 2004). In addition, a closely aligned literature explores the technology adoption decisions (McCardle 1985, Milgrom and Roberts 1990, Fine and Freund 1990, Gupta and Loulou 1998). R&D investment under technology uncertainty in a single firm setting (Balcer and Lippman 1984, Kornish, 1999) and in competitive settings (Mamer and McCardle 1987) usually yield an “*all-or-nothing*” type of policy: adopt the current best technology if the gap between current and state-of-the-art technology exceeds a certain threshold. In this paper, we will show that such “all-or-nothing” policies apply under limited conditions in startup settings to avoid bankruptcy. We illustrate that the incorporation of financial limitations in a startup setting lead to joint consideration of quantity and process investment decisions.

A recently growing body of literature deals with decision models involving the financing and operations of startups. Archibald et al. (2002) argue that if the startups are more interested in surviving than maximizing their profits, they should employ conservative strategies. On the contrary, we show that profit maximizing startups under a survival constraint could follow aggressive strategies when they have investment opportunities. Babich and Sobel (2004) provide a model to maximize the likelihood of a successful IPO for debt financed startups while Buzacott and Zhang (2004) adopt an asset based financing scheme for small and start-up firms. However, they do not explicitly model for strategic investment or competition which is central to the long term growth and survival of startups. Swinney et al. (2006) build the case on how competition between startup and established firms differs from competition between two established firms and show that a startup's preference to increase its survival affects the competition. However, they consider a single period model with a survival maximizing startup. Joglekar and Levesque (2009) analyze the distribution of venture capital between product related R&D and marketing, but do not account for either survival constraint or competition explicitly. Therefore, our research extends a growing literature on the theories of startup driven R&D and operational practices (Shane and Ulrich 2004).

Finally, an established topic of research in the entrepreneurship literature explores risk bearing as the key economic role of entrepreneurs. On one hand, Kihlstrom and Laffont (1979) and Cramer et al. (2002) show that entrepreneurs are more risk seeking, and on the other hand, Halek and Eisenhaur (2001) finds that entrepreneurs do not differ from wage earners and further, are more risk-averse than others in some cases. In a closely related empirical work, Wu and Knott (2006) study the entrepreneur's decision of market entry combined with two distinct sources of uncertainty: demand uncertainty and uncertainty regarding entrepreneur's own ability. They argue that entrepreneurs are risk averse with respect to demand uncertainty and risk seeking with respect to performance uncertainty. Recently, Corbett and Fransoo (2008) also empirically investigate whether entrepreneurs follow the newsvendor logic and how their risk preferences affect their inventory decisions. We contribute to this stream of literature by explicitly modeling for operating decisions and bankruptcy which derives the risk preferences of the firm

together with the investment opportunities, in a framework sequentially introducing technology, competition and demand risks.

In sum, the effect of cost reducing R&D on the profitability of firms has been studied extensively for established firms that are unencumbered by bankruptcy concerns. Further, cost reduction strategies adopted after the launch of a breakthrough product to maximize the profits is a relevant problem for many startup firms that take on debt and face the cash flow related threat of survival. However, this problem has not been explored formally. In the rest of this paper, we set up and study a startup's production and cost reducing investment decisions.

3. The Base Case

When making production and investment decisions, there are three key factors a typical startup considers: customer demand, startup's technological performance and competitive pressures (Shane 2007, Erzurumlu and Tanrisever 2007). To understand the interrelated impact of these factors on the operating decisions, we consider a two period model of a startup firm offering a single new product. This firm is financed by debt and must generate pre-specified level of profit after the first period to ensure survival into the second period. The objective of the firm is to maximize the total of two-period profits under the survival requirement. In this section we focus on a base case (BC) model with no demand uncertainty, and study the impact of technological performance and competitive pressures on the startup's operating decisions. In §3.1, we start with a simple model which serves as a benchmark for our analysis. Then we sequentially introduce uncertainty associated with the firm's process investment and second period competition in §3.2 and §3.3, respectively. For generality, we use the terms "return on process investment" and "technological performance" interchangeably throughout this manuscript.

3.1 A Benchmark Model

We start with some key assumptions to set up our model.

Assumption 1. *Product R&D is frozen at the beginning of the first period, i.e. at market entry.*

At least half of the startup firms in the US enter the market with a novel product (GEM Report 2007), and many of these firms continue to invest into product development effort. We do not allow for such investments, so that our analysis is not confounded by the evolution of product quality.

Assumption 2. *The startup is financed by bank loans with a constant positive interest rate.*

We consider a bank-financed startup, but our models and results trivially extend to bootstrapped startups. The interest rate is constant and positive, and upon fully paying its previous debt the startup can borrow in each period to cover its production cost and R&D investment. In general, once the loan is granted to a small firm, the loan terms including interest rate and loan limit are determined by industry practices and market conditions and do not depend on the conditions of the borrower firm (Petersen and Rajan 1994). For ease of exposure, we consider the effect of an explicit loan limit as an extension in §6. There is no time discount on the profits of the second period. The analysis is unchanged, if we consider a discount parameter between periods.

Assumption 3. *The startup goes bankrupt and gets liquidated if it cannot pay its debt at the end of each period.*

Most startups have limited access to capital markets and cannot raise additional capital other than their initial funds (Chrisman et al. 1998). In particular, informational asymmetries between the owners of the startups and the investors, and the uncertainties about the future prospects of the startup severely limit the firm's access to capital markets (Shane 2007). Hence, most new businesses are built with limited capital and face immediate bankruptcy in case of a default.

Based on these assumptions, the timing of the game is as follows. In the first period, the startup firm is a monopoly operating with a unit production cost of c_1 and receives funds, y_1 , with an interest rate of r . It allocates these funds at the beginning of the first period between production capacity, q_1 , and process R&D investment, A , which will in return linearly reduce the unit production cost in the second period to $c_2(A, \beta) = c_1 - \beta A$, where β denotes the return on investment (Gupta and Loulou 1998). At the beginning of the consecutive period, the startup realizes revenues from sales, observes reduction in unit

cost due to process investment, and makes the debt payments. In case, the revenues are not sufficient to cover the debt obligations, the firm goes bankrupt and gets liquidated. If the debt is paid in full then the firm goes into the second period and could receive a second round of funding, y_2 to invest in production, as this is the final period. We adopt a linear inverse demand function for the startup's product as $p_t(q_t) = \theta - q_t$, $t=1, 2$, where θ denotes the constant market size.

We offer the following as the benchmark model:

$$\pi_1 = \max_{q_1, A, y_1 \geq 0} (p_1(q_1) - c_1)q_1 - ry_1 - A + \pi_2(q_2; A, \beta)$$

subject to $c_1 q_1 + A \leq y_1$ (1a)

$$(p_1(q_1) - c_1)q_1 - ry_1 - A \geq 0$$
 (1b)

where

$$\pi_2(q_2; A, \beta) = \max_{q_2, y_2 \geq 0} (p_2(q_2) - c_2(A, \beta))q_2 - ry_2$$

subject to $c_2 q_2 \leq y_2$ (1c)

In this model, (1a) and (1c) represent the financial constraints in the first and second periods, respectively, such that the total expenditures of the firm in each period are limited by the amount of money borrowed. (1b) denotes the *survival* constraint requiring that the money borrowed in the first period should be paid back with interest at the end of the period. Based on our model assumptions, (1a) and (1c) must be binding. Therefore, we re-state (1) as follows:

$$\pi_1 = \max_{q_1, A \geq 0} (p_1(q_1) - (1+r)c_1)q_1 - (1+r)A + \pi_2(q_2; A, \beta)$$

subject to $(p_1(q_1) - (1+r)c_1)q_1 - (1+r)A \geq 0$ (2)

where $\pi_2(q_2; A, \beta) = \max_{q_2 \geq 0} (p_2(q_2) - (1+r)c_2(A, \beta))q_2$

Assuming that the return on investment β is constant and equal to μ , we characterize the operating decisions and profits in Proposition 3.1. We use the superscript *bc* to denote the benchmark case. Later we will use *c* for the case with competition and *t* for the case with uncertain return on

investment. (See the Appendix for proofs of the lemmas, propositions and corollaries unless stated otherwise.)

Proposition 3.1 *When demand and the return on investment are deterministic, the startup's total profits*

are maximized at the monopoly quantity, $q^ = q_m = \frac{\theta - (1+r)c}{2}$. Following, the process R&D investment*

of the startup is bounded by the discounted monopoly profits, $A_{\max} = \frac{\pi_m}{(1+r)} = \frac{(\theta - (1+r)c)^2}{4(1+r)}$. Assuming

that the cost cannot be driven to zero, the optimal process investment for the startup, A^ , and the optimal expected profits, π^* , are given by the following:*

$$A^* = \begin{cases} A_{\max} & \text{if } \Delta^{bc} \geq 0 \\ 0 & \text{o/w} \end{cases}, \text{ and } \pi^* = \begin{cases} \left(\frac{\theta - (1+r)(c - \mu A_{\max})}{2} \right)^2 & \text{if } \Delta^{bc} \geq 0 \\ \frac{(\theta - (1+r)c)^2}{2} & \text{o/w} \end{cases}$$

where $\Delta^{bc} = \mu^2(\theta - (1+r)c)^2 + 8\mu(\theta - (1+r)c) - 16$.

A close examination of (2) reveals that the startup's optimization problem is partially separable in production quantity and process investment. Consequently, we find in Proposition 3.1 that it is optimal to produce the monopoly quantity, and the monopoly profits limit the investment amount due to the survival constraint.

To explain the investment decision, we define the firm's propensity to invest in process improvement as Δ . In particular, if $\Delta < 0$ then, the firm does not invest in process improvement. Therefore, the profits in each period are identical and equal to the monopoly profits. However, if the firm's propensity to invest is sufficiently high, $\Delta \geq 0$, then it would allocate all of its funds in process improvement and make zero net profits after the debt payments, in the first period. It could later generate enough revenues with the second period sales to compensate for the missed earnings of the first period. In particular, the firm either chooses not to invest, $A^* = 0$, or if it chooses to invest, it invests the maximum possible amount, A_{\max} , which would maximize its profits without going bankrupt. Therefore, the optimal

process investment decision can be characterized by an *invest-all-or-nothing* threshold policy. Further, as the mean return on investment and market size increase, the firm's propensity to invest also increases.

3.2 Technology Uncertainty

So far, we have assumed that process investment reduces the future unit cost of the firm by a deterministic amount. Nevertheless, for startups with niche processes like Faradox and BigFoot Networks return on process investment is inherently uncertain. To take this into consideration, we extend our discussion in the benchmark case to consider the impact of the return on investment uncertainty on the startup's operating decisions and profits. In particular, for every dollar invested, we assume that the second period cost is reduced by a random amount described by $\tilde{\beta}$, with a known distribution function, $\psi(\tilde{\beta})$, with a mean of μ and a variance of σ^2 . The following proposition characterizes the optimal production and investment decisions in the benchmark model with technology uncertainty.

Proposition 3.2 *When demand is deterministic, but the return on investment is uncertain, then the startup's optimal production quantity is equal to the monopoly quantity. And, the optimal process investment and profits are, respectively, given by*

$$A^* = \begin{cases} A_{\max} & \text{if } \Delta^m \geq 0 \\ 0 & \text{o/w} \end{cases}, \text{ and } \pi^* = \begin{cases} \frac{(\theta - (1+r)(c - \mu A_{\max}))^2 + (1+r)^2 \sigma^2 A_{\max}^2}{4} & \text{if } \Delta^m \geq 0 \\ \frac{(\theta - (1+r)c)^2}{2} & \text{o/w} \end{cases}$$

where $\Delta^m = (\sigma^2 + \mu^2)(\theta - (1+r)c)^2 + 8(\theta - (1+r)c)\mu - 16$.

With return on investment uncertainty the firm's propensity to invest becomes larger than the case with no uncertainty in return on investment, i.e., $\Delta^m = \Delta^{bc} + (\theta - (1+r)c)^2 \sigma^2$. Further, the firm profits are also non-decreasing in σ^2 , so when selecting among production technologies, the startup prefers technologies with more variable return compared to the ones with relatively certain returns. This may seem like a counterintuitive result, but if this technology adoption proves to be successful, then the firm could obtain significant cost reduction and have a major increase in profits. That is, the startup disproportionately benefits from upside deviation in return on process investment. In our model, this is

driven by the convex monopoly profits, $(\theta - (1+r)c)^2 / 4$, with respect to the unit cost in the second period.

3.3 Competition

In this section, we study the case with competition. The sequence of events is precisely the same as the case without competition. The difference is that at the beginning of the second period a competitor with an identical product enters the market and firms play a Cournot game where the competitor's best response quantity is denoted by q_c . The updated sequence of events and the startup's decisions are summarized in Figure 1.

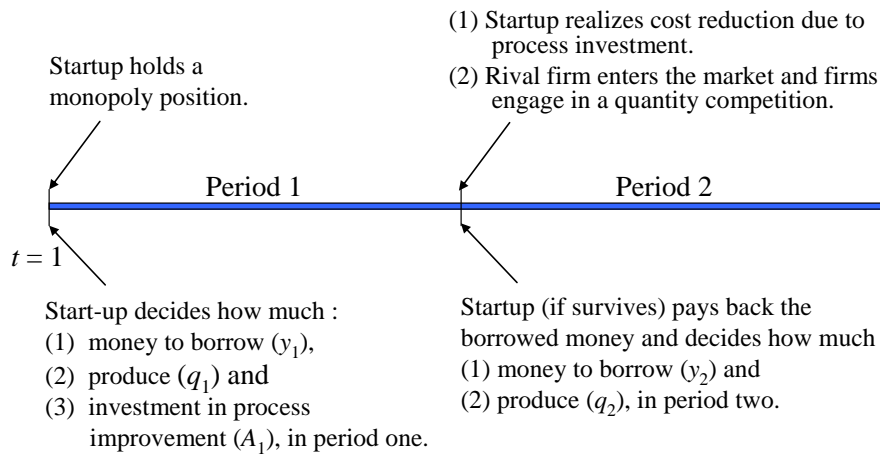


Figure 1: Sequence of Events and Decisions in a Two Period Model with Competition

When a competitor is to enter the market, the startup may not fully know the entrant's production system for a new product, but it may know the competitor's cost through a probability distribution function. Indeed, Faradox Inc., a producer of high energy-density capacitors, mentioned in our interview that there was tremendous amount of theoretical research in the field of capacitor technologies and it was likely that someone might enter their market by developing a new process to produce high energy-density capacitors. Hence, from the perspective of Faradox, the efficiency of the prospective competitor in the future is highly uncertain and exogenous.

To incorporate this into our benchmark model, we assume that the unit variable cost of the competitor, $\tilde{\xi}$, is distributed with a probability density function of $\phi(\tilde{\xi})$, and has a mean of λ and a variance of τ^2 . For ease of exposure, we exclude technology uncertainty in this section, but our findings here also trivially extend to the case with both technology uncertainty and competition. The following proposition characterizes the startup's optimal production and investment decisions for the benchmark model with competition.

Proposition 3.3 *Under deterministic demand and return on investment, when there is competition in the future period, then the startup's optimal production quantity is equal to the monopoly quantity. The optimal process investment and the optimal profits are, respectively, given by*

$$A^* = \begin{cases} A_{\max} & \text{if } \Delta^c \geq 0, \\ 0 & \text{o/w} \end{cases}, \text{ and}$$

$$\pi^* = \begin{cases} \frac{1}{9} \{ (\theta - 2(1+r)c + \lambda)^2 + 4A_{\max} \{ \mu(\theta - 2(1+r)c + \lambda) + \mu^2 A_{\max} \} + \tau^2 \} & \text{if } \Delta^c \geq 0 \\ \frac{(\theta - (1+r)c)^2}{4} + \frac{1}{9} \{ (\theta - 2(1+r)c + \lambda)^2 + \tau^2 \} & \text{o/w} \end{cases},$$

where $\Delta^c = (\mu(\theta - (1+r)c) + 2)^2 - 4\mu((1+r)c - \lambda) - 13$.

From Proposition 3.3, we observe that the propensity of the startup to invest increases with the expected unit cost of the competitor, i.e., $\partial\Delta^c / \partial\lambda > 0$. In other words, when faced with a strong competitor, the startup is less willing to invest since the benefits of investment is reduced under competition. According to our investment policy, the variance of the competitor's cost would have no effect on the investment decision so long as the quantity response function is linear in the realization of competitor's cost. However, since Cournot profits are convex in the competitor's cost, the optimal profits increase as the strength of competition gets more variable because the startup disproportionately benefits from high cost entrants.

Comparing the firm's propensity to invest with and without competition for various levels of competitor's unit cost, we can further explain the impact of the strength of future competition on the firm's propensity to invest:

Corollary 3.4 *In the presence of competition the startup's propensity to invest increases compared to the benchmark case, if the expected competitor is relatively weak. In particular:*

$$\begin{aligned} i) \Delta^{bc} \geq \Delta^c & \quad \text{if} \quad \lambda \leq \theta - 7/4\mu, \\ ii) \Delta^{bc} \leq \Delta^c & \quad \text{if} \quad \lambda \geq \theta - 7/4\mu. \end{aligned}$$

Corollary 3.4 shows that the firm may find it optimal to invest in the presence of competition when it is better off with no investment in the benchmark case, i.e., $\Delta^{bc} < 0 < \Delta^c$. Therefore, the shadow of future competition may encourage investment by the startup depending on the expected strength of the competitor.

4. The Stochastic Demand and Survival Case

In the BC, we studied the startup firm's operating decisions under deterministic demand. However, in most cases the startup would have very limited information about the demand, especially for a brand new product. In this section, we examine our model with stochastic demand in each period, and replace the deterministic survival constraint of the BC with a probabilistic survival requirement.

4.1 The Model with Stochastic Demand and Survival

In this case, we assume a demand shock, $\tilde{\varepsilon}_t$, in each period t ($t = 1, 2$) with a normal probability density function, $\varphi(\cdot)$, with mean zero and variance v^2 , and cumulative distribution function, $\mathcal{G}(\cdot)$. The minimum profit level required for the survival denoted by $\underline{\pi}$ is exogenous and includes the overhead costs like rents and wages. We define the first period net expected monopoly profits, $\pi_m - \underline{\pi}$, as the immediate economical viability of the firm (Note that the expected monopoly profits is given by

$\pi_m = E[(p(q_m, \tilde{\varepsilon}_1) - (1+r)c)q_m]$. Under the SDSC case with competition, technology uncertainty and stochastic demand, the two-period expected profit maximization problem of the startup is given as:

$$\begin{aligned}
z^* &= \max_{q_1, A} E_{\tilde{\varepsilon}_1} \left\{ (p(q_1; \tilde{\varepsilon}_1) - c_1)q_1 - r(c_1 q_1 + A) - A \right\} + E_{\tilde{\xi}, \tilde{\beta}} \pi_2(A; \varepsilon_1, \tilde{\xi}, \tilde{\beta}) \\
\text{s.t.} \quad & q_1, A \geq 0 \\
& \text{where} \\
\pi_2(A; \varepsilon_1, \xi, \beta) &= \max_{q_2, I} E_{\tilde{\varepsilon}_2} \left\{ (p_2(q_2 + q_c, \tilde{\varepsilon}_2) - (1+r)c_2(A, \beta; A_0))q_2 \right\} \\
\text{s.t.} \quad & \left. \begin{aligned} \pi_1(\varepsilon_1) - \underline{\pi} &\geq M(I-1) \\ q_2 &\leq MI \\ q_2 &\geq 0, I \in \{0, 1\} \end{aligned} \right\} \text{ Stochastic survival constraints}
\end{aligned} \tag{3}$$

where I is the first period survival indicator (= 1 if the firm survives the first period) and M is a large number. In (3), the constraints in the second stage of the problem only hold if the startup has survived the first period. In particular, unless the first period profit for the startup meets the minimum level required for survival ($\pi_1 < \underline{\pi}$), the startup cannot play the second period quantity game. In this case, the survival indicator variable I in (3) has to be zero and consequently, q_2 is also forced to zero.

We solve the optimization problem in (3) by backward induction. In the second period, firms play a Cournot game to maximize their expected profits. Hence, the startup's equilibrium profit, if it could play the second period game, is given by $\pi_2(A; \varepsilon_1, \xi, \beta) = \left((\theta + \bar{\varepsilon}_2) + \xi - 2(1+r)c_2(A, \beta) \right)^2 / 9$ where $E(\varepsilon_2) = \bar{\varepsilon}_2$. By assumption, $\bar{\varepsilon}_2 = 0$. For the startup to participate in the second period, first it has to survive in the first period only if $\pi_1(\varepsilon_1) \geq \underline{\pi}$. After substituting the optimal second period solution, the startup's problem in (3) becomes

$$z^* = \max_{q_1, A \geq 0} E_{\tilde{\varepsilon}_1} \left\{ (p(q_1, \tilde{\varepsilon}_1) - c_1)q_1 - r(c_1 q_1 + A) - A \right\} + E_{\tilde{\xi}, \tilde{\beta}, \tilde{\varepsilon}_1} \left\{ \frac{(\theta + \tilde{\xi} - 2(1+r)c_2(A, \tilde{\beta}))^2}{9} \mid \pi_1(\tilde{\varepsilon}_1) \geq \underline{\pi} \right\} \tag{4}$$

Unlike the BC, the maximization problem is not separable in production and investment decisions, and it is non-convex. Nevertheless, we can still prove the following important relationship for the optimal decisions.

Proposition 4.1 *When demand is stochastic, the startup firm in the first period either adopts a conservative operating policy by producing and investing less than the monopoly levels i.e.,*

$$q^* \leq q_m = \frac{\theta - (1+r)c}{2}, \quad A^* \leq A_m = \frac{\pi_m - \pi}{1+r},$$

or an aggressive operating policy by producing and investing more than the monopoly levels, i.e., $q^ \geq q_m, A^* \geq A_m = \frac{\pi_m - \pi}{1+r}$.*

Proposition 4.1 provides an interesting risk based justification linking production and investment decisions of a startup under stochastic demand and bankruptcy risk. The firm is aggressive in investment decision ($A \geq A_m$), if and only if it is also aggressive in production ($q^* \geq q_m$). Or, the firm is conservative in investment decision ($A < A_m$) if and only if it is also conservative in production ($q^* < q_m$). If an aggressive investment is planned, then the expected cash flows under the monopoly production plan is not sufficient to cover the debt payments. Hence, the firm increases its production quantity above the monopoly level so as to benefit from upside demand realizations and to increase its survival chances and conversely, a conservative investment reduces production below the monopoly level.

In the following proposition, we establish the intimate connection between the optimal operating policy of the firm and the survival probability.

Proposition 4.2 *An optimal operating policy is aggressive (conservative) if and only if its survival*

probability, $P \square 1 - \mathcal{G}_{\varepsilon_1} \left(\frac{\pi + (1+r)A}{q} + q - \theta + (1+r)c \right)$, is less (more) than fifty percent.

Proposition 4.2 provides an equivalent survival-based definition for optimal aggressive and conservative operating decisions. That is, optimal operating policies that survive less (more) than 50%

chances always involve producing and investing more (less) than the monopoly levels, and vice versa. This implies that an aggressive firm is expected to go bankrupt on average while a conservative firm is expected to survive. In general, an operating policy is considered to be riskier as the survival probability decreases.

In the reminder of this section and in §5, we explore the factors that drive the optimal operating decisions of the startup under stochastic demand. In Proposition 4.1 we implicitly assume that the startup would find an investment opportunity. However, that may not be the case. Corollary 4.1 considers the impact of the existence of investment opportunities (with positive NPV) on the operating decision of economically viable startup firms. Recall that immediate economical viability means the firm's first period net expected monopoly profits are non-negative.

Corollary 4.1 *Suppose the startup firm is immediately economically viable in the first period, i.e., $\pi_m - \underline{\pi} > 0$, then*

i) If there are no process investment opportunities, $A = 0$, the firm always adopts a conservative operating policy. That is, the firm produces less than the monopoly quantity.

ii) If there is an opportunity for process investment, $A \geq 0$, then the startup may either adopt an aggressive or conservative operating policy.

According to Corollary 4.1, when there are no investment opportunities, immediately economically viable startups always choose a conservative operating policy. To better illustrate our finding, we examine a simple situation with no minimum level of profits, $\underline{\pi} = 0$ and we let the demand shocks in each period (ε_t for $t = 1, 2$) be uniformly distributed with $U[-b, +b]$. Then, the optimization problem takes the following form:

$$\max_{q \geq 0, A \geq 0} f(q, A) = [(\theta - q)q - (r + 1)(cq + A)] + \left[\frac{\tau^2 + (\theta + \lambda - kc)^2 + 2(\theta + \lambda - kc)k\mu A + k^2(\mu^2 + \sigma^2)A^2}{9} \right] \left[\frac{b - \left(\frac{(1+r)A}{q} + (1+r)c + q - \theta \right)}{2b} \right]$$

When there are no investment opportunities ($A = 0$), $f(q, 0)$ is concave in q and the optimal quantity is

given by $q^* = \frac{\theta - (1+r)c}{2} - \frac{1}{4b} \left(\frac{(\theta + \lambda - 2(1+r)c)^2}{9} \right) < q_m$, which agrees with our finding that in the

absence of investment opportunities, the firm always behaves conservatively. The positive second term of the optimal quantity, q^* , above represents the under-production amount due to stochastic bankruptcy risk in order to increase the probability of survival. In particular, if the bankruptcy risk is to be removed from the decision framework, the firm simply produces the first best production level, i.e., the monopoly quantity. That is, the bankruptcy risk drives an economically viable startup to adopt a conservative policy in the absence of investment opportunities. In addition, startups with high expected future prospects, such as a large market base, an already efficient process technology or a relatively weak competitor, focus more on survival in anticipation of future profits. That is, they deviate more from their first best operating plans and choose a more conservative policy. We will numerically investigate the optimal operating policies for this case in §5.

We now turn our attention to a firm that is not economically viable in the first period. We implicitly assume that the firm is economically viable over the planning horizon. Otherwise, it is optimal to liquidate the firm at time zero.

Corollary 4.2: *Suppose the startup firm is not immediately economically viable in the first period, i.e., $\pi_m - \underline{\pi} \leq 0$, then the firm always adopts an aggressive operating policy, regardless of the existence of investment opportunities.*

When the firm is not immediately viable, e.g., due to high operating costs relative to immediate profits, its survival is contingent on the upside deviations in market demand. To benefit from these upside deviations and survive, the firm should increase its production above the monopoly quantity. Consequently, even with no investment opportunities the firm would always choose an aggressive operating policy. We summarize the effects of the immediate economical viability and investment opportunities on the operating policy of the startup in Table 1.

Our results show that an immediately viable startup with investment opportunities may either adopt a conservative or an aggressive operating policy depending on the market parameters. To further investigate this case and the impact of market parameters on the optimal operating decisions, we present a comprehensive computational analysis in the next section. We also note that this case is not analytically tractable. There are several reasons for this, including that the objective function in (4) is neither jointly convex nor concave in q_1 and A for all feasible set of parameter settings.

Table 1: Operating Policy of the Startup

	Immediately Viable Startup ($\pi_m - \underline{\pi} > 0$)	Immediately Non-viable Startup ($\pi_m - \underline{\pi} > 0$)
With Investment Opportunities	Conservative or aggressive operating policy depending on market parameters	Aggressive operating policy
With No Investment Opportunities	Conservative operating policy	Aggressive operating policy

4.2 Computational Analysis

In this section, we focus on a set of numerical analyses to illustrate the impact of key market factors (demand uncertainty, technological performance, competition and minimum required level of profits), on the optimal operating policies (production and process investment) of the immediately viable startups with investment opportunities. We also provide insights that link the BC to SDSC.

4.2.1 Design of Numerical Experiments

Our design of experiment focuses on the optimal survival probability as the relevant measure of the risk taken by the firm. The survival probability is an endogenous variable determined by the firm's production and investment policy. Recall that a conservative policy survives with probability more than 50% while an aggressive strategy bankrupts with probability more than 50%. And, a conservative (aggressive) policy involves producing less (more) than the monopoly quantity and investing less (more) than the expected net monopoly profits.

We begin with examining the impact of mean return on investment in §4.2.2 in an experimental setup that has no competition, deterministic return on investment and zero interest rate as in BC. In this case the optimization problem in (4) reduces to

$$\max_{q, A \geq 0} f(q, A) = [(\theta - q)q - cq - A] + \frac{(\theta - (c - \mu A))^2}{4} \left(1 - F_{\varepsilon_1} \left(\frac{\pi + cq + A}{q} + q - \theta \right) \right)$$

Following, we explore the impact of technological uncertainty and competition on the operating decisions in §§4.2.3 and 4.2.4, respectively. For ease of exposition, throughout our numerical analysis we fix the market size and initial unit cost ($\theta = 10$, $c = 7$), so $q_m = 1.5$ and $\pi_m = 2.25$. The standard deviation of demand shock is set to $v = 1.2$. The impact of different levels of v is investigated in §4.2.5. We present a selective set of our results, but we have tested and confirmed similar results with entire sets of values that the model parameters can take.

4.2.2 Benchmark Case with Stochastic Demand

In this section we examine the impact of mean return on investment, μ , and the immediate economical viability of the firm through minimum required profits, π . Figure 2 illustrates the optimal operating decisions and the associated survival probabilities for all reasonable levels of μ . We observe in Figure 2 that it is optimal to invest if $\mu > 0.9$. Then, as in the BC, a threshold type of investment policy is optimal, but the policy does not have an invest-all-or-nothing structure. As the firm's potential efficiency in cost reduction increases, the firm raises its investment amount in process technology leading to riskier operating decisions with lower survival probability. Indeed, aggressive investment becomes the optimal policy for $\mu > 2.3$. In addition, the production quantity may either increase or decrease with the investment level to create an operational hedge in response to optimal investment decision.

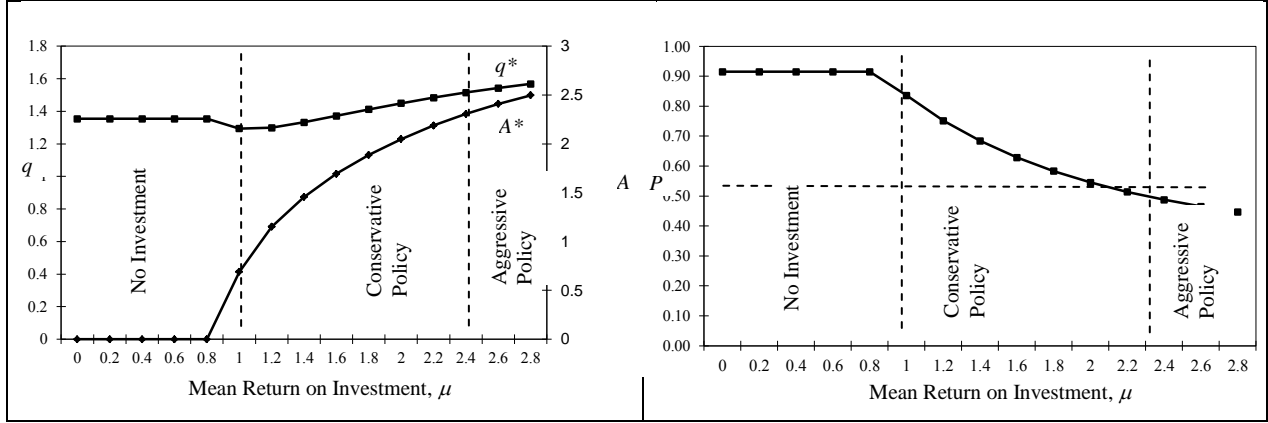


Figure 2: Optimal Operating Decisions and Survival Probability as a Function of μ for $\underline{\pi} = 0$

Figure 3 illustrates the interactive impact of μ and $\underline{\pi}$ on the optimal policy. We observe that ‘no investment’ region expands as $\underline{\pi}$ increases. In particular, when $\underline{\pi}$ is high, investment creates a very high bankruptcy risk consuming the limited short-term profits of the firm. Therefore, the firm avoids investment. On the hand, if $\underline{\pi}$ is low, then the firm is expected to have cash in the future and, it may invest some of this cash in process improvement without diminishing its survival probability. Further, depending on its mean return on investment, the optimal policy is either conservative or aggressive.

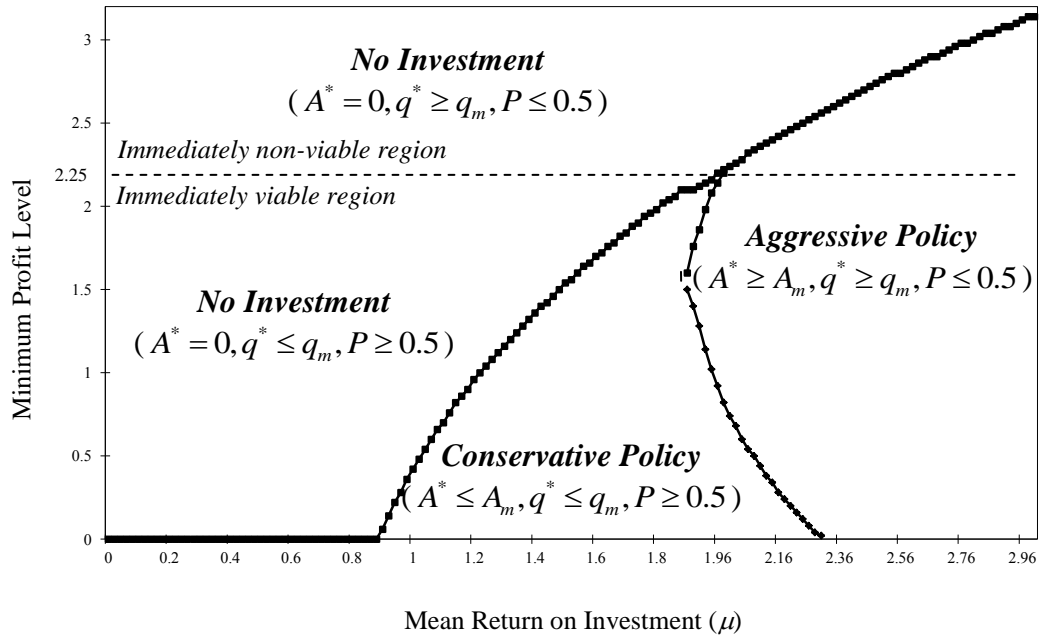


Figure 3: Interaction of μ and $\underline{\pi}$ under SDSC Case

4.2.3 Technology Uncertainty

In the previous section we discuss the impact of mean return on investment on the operating decisions of the firm, but we did not examine the associated uncertainty. We complete this discussion by illustrating the effect of technology uncertainty, σ , in Figure 4. For ease of discussion, in the remainder of this section we set $\underline{\pi} = 0$, but similar results can be obtained for other values. Under deterministic demand, from Proposition 3.2, we know that as the return on investment gets more variable and the chances of upside deviations increase, the firm is more willing to invest. Similarly, when demand is uncertain, Figure 4 leads to the observation that for a given level of μ , an increase in technology uncertainty decreases the survival chances of the firm, by inducing more aggressive operating decisions with higher production and investment levels.

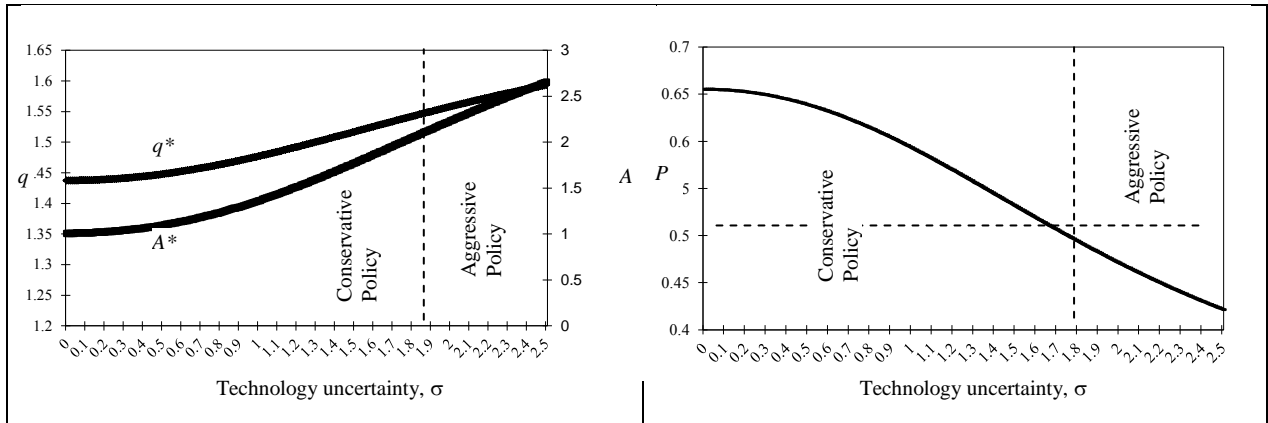


Figure 4: Optimal Production Quantity and Process Investment as a Function of σ ($\mu = 1.5$)

4.2.4 Competition

In this section we explore the impact of competition on the operating decision of the firm under stochastic demand and deterministic return on investment. In Figure 5 we present the optimal operating decisions as well as the associated survival probabilities as the competitor's expected cost λ changes for a given level of return on investment μ .

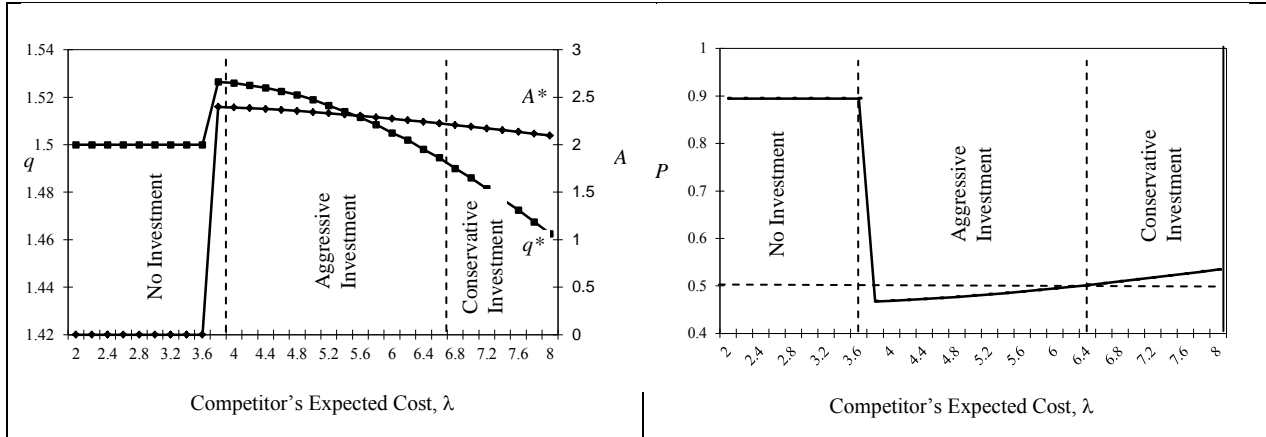


Figure 5: Optimal operating decisions and survival probability as a function of λ ($\mu = 1.5$).

Similar to the BC, Figure 5 shows that the firm starts investing when the competitor is sufficiently weak and benefits from investment in the future period. However, the firm may invest (and produce) either aggressively or conservatively depending on the level of λ . Further, for a given level of μ it does not necessarily keep raising its investment amount as the competition gets weaker because although the expected marginal second period profit of the firm is increasing with λ , a higher investment amount also increases firm's exposure to bankruptcy. Figure 5 illustrates this tradeoff that firm chooses a conservative investment policy when faced with very weak competitors to control the bankruptcy risk.

Figure 6 further explores the interrelated impact of competition and the mean return on process improvement on the optimal operating policy of the firm. The startup makes no process investment if it is not efficient to engage in competition with a relatively strong competitor. A conservative policy is chosen when the future entrant would not intensify competition because it has relatively high cost production process. Further, an aggressive policy is adopted when the startup is sufficiently efficient in cost reduction and the competitor is neither too strong nor too weak. In this case, the second period profits are distributed more equally between the firms. Hence, by following an aggressive strategy (if it is not too costly) the startup may significantly increase its share of expected profits in the second period and obtain a strong future market position.

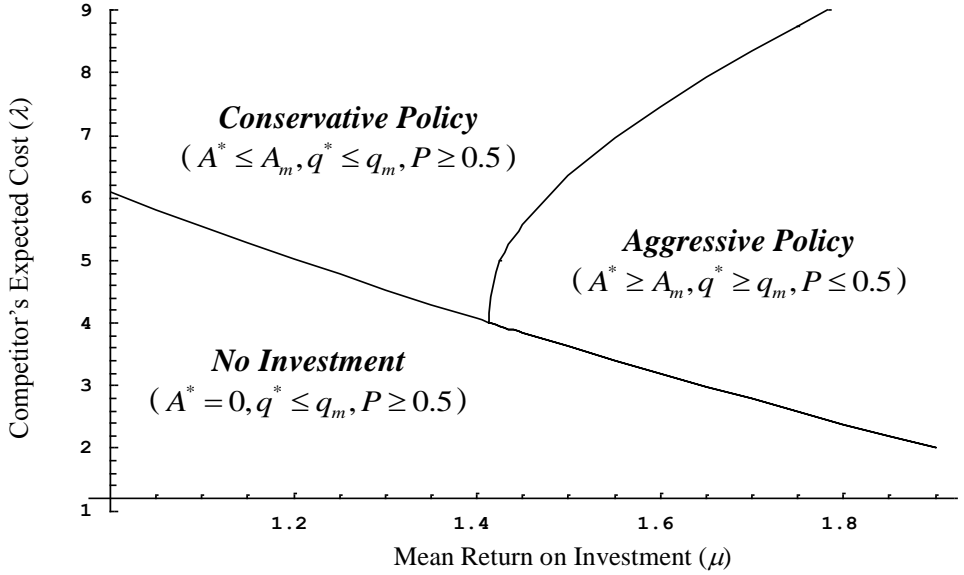


Figure 6: Interaction of $\mu - \lambda$ under SDSC Case

4.2.5 Demand Uncertainty

Demand uncertainty is an exogenous factor influencing the bankruptcy risk. Recall that with demand uncertainty investment amount may be either less or more than monopoly investments. Figure 7 illustrates that demand uncertainty shrinks investment regions when it is optimal to start investing in process R&D, and the thresholds for process investment in the SDSC are higher than the BC, ceteris paribus.

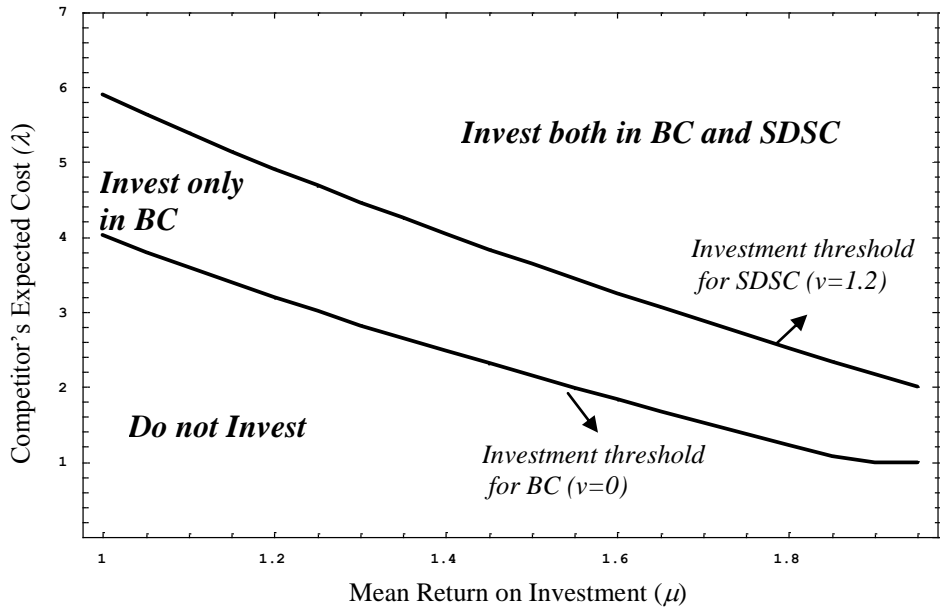


Figure 7: Partitioning of the Process Investment Space under the BC and SDSC

Figure 8 shows the impact of demand variability and mean return on investment on the optimal policy when there is no competition and technological variability. As shown in Figure 8, when v is very low, the startup either chooses not to invest or invests conservatively. A higher variability of demand provides the firm with the opportunity of survival under aggressive investment plans. Hence, aggressive policies are only possible if demand is sufficiently variable to provide high demand and the firm is efficient in cost reduction. Indeed, when demand is deterministic as in the BC, aggressive policies are infeasible. These observations combined with our earlier findings support that demand variability is necessary to induce immediately viable firms to increase their investment amount and adopt aggressive policies if the increased second period profits due to aggressive investment compensate the excess risk taken by the firm. Also note that Figure 8 generalizes Figure 2 which is constructed for $v=1.2$ only.

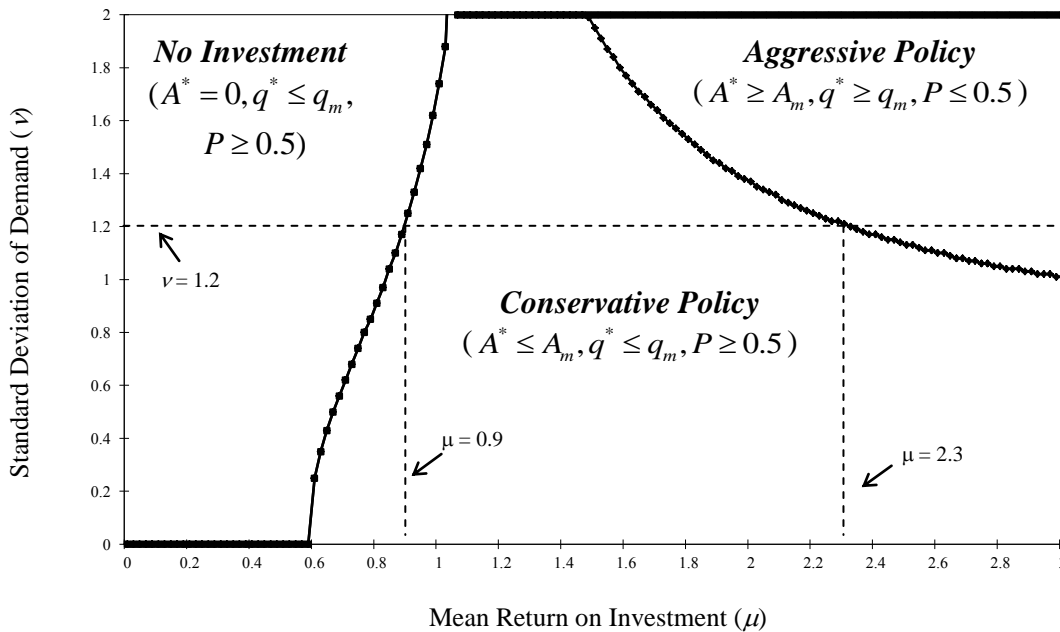


Figure 8: Interaction of $\mu-v$ under SDSC Case

5. Debt Capacity

To isolate the impact of bankruptcy risk, we have assumed throughout the paper that the startup firm is able to borrow enough to finance its optimal operating policy in the first period. In this section we

introduce a debt capacity, L , which limits the total cash available to the firm ($cq + A \leq L$), and examine its effect on the risk preferences of the startup. In Proposition 5.1 we characterize the impact of debt capacity on the base case results under deterministic demand.

Proposition 5.1 *Under deterministic demand and return on investment, with no future competition,*

i) If $L \geq cq_m + A_{\max}$, the debt capacity is never binding.

ii) If $L \leq cq_m$, the debt capacity is always binding.

iii) If $cq_m < L < cq_m + A_{\max}$, the debt capacity may or may not be binding depending on the market parameters.

If the debt capacity is larger than the maximum amount of cash that may be needed by the firm, i.e., $L \geq cq_m + A_{\max}$, additional cash has no value to the firm. In this case, the optimal operating decisions are characterized by Proposition 3.1 and the firm's propensity to invest is unaffected. However, if debt capacity is not sufficient to finance the monopoly production level, the firm may invest additional capital into production and increase profits. Also, when the debt capacity is moderately tight ($cq_m < L < cq_m + A_{\max}$), the firm may benefit from additional cash if investment is optimal when there is no debt capacity. Further, in the following corollary we discuss the firm's propensity to invest with a binding debt capacity.

Corollary 5.2 *Under deterministic demand, when there is a binding debt capacity, startup's propensity to invest decreases.*

We show (Proposition 3.1) that the startup's operating policy with no debt capacity can be described as an "all-or-nothing" policy, i.e., whether to invest nothing or to invest all of the net monopoly profits, A_{\max} . However, under a binding debt capacity, the startup can never finance to invest as much as A_{\max} . Besides, since the marginal return on investment is increasing in A , reducing the maximum investment level decreases the benefits of scale economies in investment and hence, decreases the firm's

propensity to invest. Figure 9 presents the impact of debt capacity on the optimal operating policy of the startup under stochastic demand and survival.

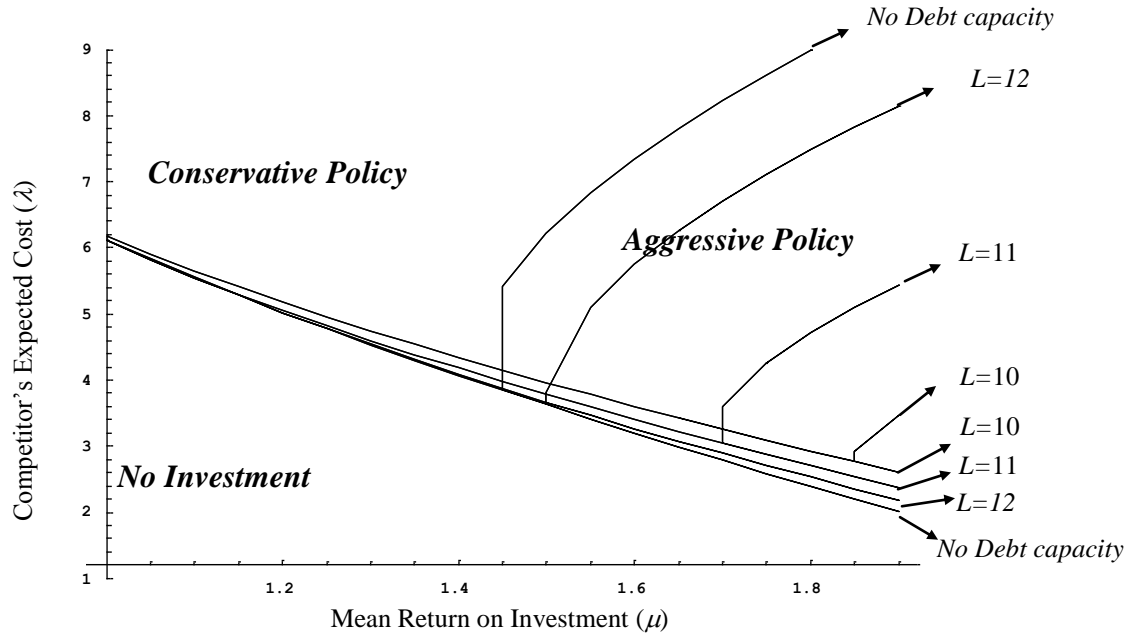


Figure 9: Impact of Debt Capacity on the Operating Policy in SDSC

Figure 9 is identical to Figure 6 for no debt capacity case. We observe that, our discussion in §4.1.4 still holds, but the limiting effect of a tighter debt capacity is clear. Aggressive and conservative policy regions shrink and no-investment region expands with a tighter debt capacity. Overall, our observations suggest that the firm's propensity to invest is reduced with the debt capacity in the deterministic demand case, and the debt capacity makes the firm more conservative under stochastic demand. However, the basic results we have shown for the BC and SDSC remain valid under reasonably tight debt capacities.

6. Discussion and Concluding Remarks

Existing organizational theories (Bhide 2000) have marshaled evidence to argue that startup managers make myopic choices in their long term investment decisions when faced with uncertainty and financial pressure. Our analysis explores the impact of three key risk drivers (demand, technology and competition) on the short and long term production and process investment decisions of startups under the presence of

explicit financial constraints. Since financial limitations alter optimal operating decisions; our results provide a risk based justification for startups linking their production with their process R&D investment.

6.1 Optimal Operating Decisions of Startups with Deterministic Demand

Under deterministic demand we find that the startup always produces the monopoly quantity and uses a process investment threshold policy involving an “invest-all-or-nothing” type of structure. The investment policy is described by the firm’s propensity to invest. We investigate the impact of demand, technological performance and competition on the firm’s propensity to invest in process improvement. In a large market the firm has high potential to recover the process investment. Similarly, higher expected return on investment (better expected technological performance) increases the potential benefits of investment and makes the firm more willing to invest. Further, as in new technology development, the firm disproportionately benefits more from upside deviations of return on investment. Hence, the firm’s propensity to invest increases as the process technological performance gets more variable.

Impact of competition is more involved. As the expected competitor in the future gets stronger, it chips off future profits and the startup’s propensity to invest decreases. However, compared to the monopoly situation, the startup may invest to mitigate the impact of competition and secure its future earnings if the competitor is not very strong. We summarize the impact of key parameters on the optimal process investment of the startup under deterministic demand (base case) in Table 2. Recall that in BC the firm always produces the monopoly quantity.

Table 2: Impact of Key Factors on the Optimal Operating Policy under Deterministic Demand

Factor	Impact on Optimal Operating Policy
Return on process investment	The firm’s propensity to invest Δ increases with mean (μ) and standard deviation (σ) of return on process technology investment.
Competition	Δ decreases with expected level of competition (λ). Δ increases compared to the monopoly case, if the competitor is not very strong.
Debt Capacity	Δ decreases with debt capacity (L).

6.2 Optimal Operating Decisions of Startups with Uncertain Demand

In the deterministic base case the monopoly firm's profitability is guaranteed after the first period. However, in the stochastic demand and survival case, demand may be too low and profitability is not assured. Therefore, the probabilistic survival constraint becomes critical. Recall from §2 that this constraint is a unique risk driver that has not been addressed in literature. It shapes our results as follows: while the decision to invest in process development at early stages reduces the startup's profits and increases its exposure to bankruptcy, the low cost production process in subsequent periods could improve the startup's competitiveness and its market position. Hence, our core result states that under stochastic demand a central consideration in the startup's decision on the investment allocation is the tradeoff between the long-term expected profits and short-term bankruptcy risk.

When there is no demand risk, the firm always produces the monopoly level and invests nothing or all of the monopoly profits, as shown in the base case. When faced with stochastic bankruptcy risk, the startup sacrifices some short-term profits by deviating from its first best production plan. In the conservative case, the firm under-produces so as to allocate more cash to process R&D while controlling the survival chances. Further, depending upon competitor's cost, technological performance and aggregate demand, the startup may also invest aggressively by increasing the investment level above the BC level. In this case, the startup over-produces (more than the monopoly quantity) to cover the higher bankruptcy risk due to the aggressive process investment. That is, from an operational perspective, the startup hedges aggressive investment decisions by producing aggressively while conservative investment decisions involve conservative production plans.

Further, we identify two factors influencing operating decisions of startups: (1) the existence of positive NPV investment opportunities and (2) the immediate economical viability of the firm. We have shown that the startups that are not immediately economically viable adopt aggressive business plans, while immediately economically viable startups with no investment plans would always be conservative. Under stochastic demand, an intriguing case for decision making is revealed when there exists investment

opportunities, *and* the firm is immediately economically viable. In this case, we numerically investigate the startup’s optimal operating decisions and find that depending on demand uncertainty, success in process development, and anticipated competition, the firm could either follow an aggressive or conservative investment strategy. Our analysis indicates that the startup becomes aggressive and adopts riskier operating plans with lower survival chances when its efficiency of cost reduction increases or when faced with moderately strong competitors. Our results also demonstrate that demand uncertainty drives aggressive behavior. Since the survival of aggressive policies is dependent on the upside realizations in demand, highly variable markets create an incentive to follow aggressive policies. These results are summarized in Table 3.

Table 3: Impact of Key Factors on the Optimal Operating Policy under Stochastic Demand

Factor	Impact on Optimal Operating Policy
Return on process investment	No investment for low returns Conservative operating policy with moderate returns Aggressive operating policy with sufficiently high μ and σ
Immediate economical viability	No Investment for low levels of immediate economical viability, $\pi_m - \underline{\pi}$. Conservative operating policy for high levels of $\pi_m - \underline{\pi}$ Aggressive operating policy for moderate levels of, $\pi_m - \underline{\pi}$
Competition	No Investment with strong competitors Conservative operating policy with weak competitors Aggressive operating policy with moderately strong competitors
Demand	Aggressive operating policy with higher demand variability
Debt capacity	Conservative operating policy with tighter debt capacity

We discussed our results with startup managers in search of process improvement opportunities in order to seek feedback about the model outcomes. In general, there seems to be an agreement among our respondents about the risky nature of process investment and the operational levers for hedging these risks. Some managers also pointed to additional factors that come into play into such decisions. For instance, the managers at Bigfoot Networks indicated that they currently outsourced semiconductor

manufacturing and was looking for in-sourcing options, because it might provide opportunities for cost reduction. Faradox decided to use an emerging technology fund from Central Texas Regional Center of Innovation and Commercialization to develop a new fabrication process because production process has become a key part of their long-term business model. Below, we discuss the limitations of our model and potential extensions that have come up as a result of such field work.

6.3 Limitations and Extensions

We have studies process improvement which reduces the future unit cost. However, it would be interesting to study other forms of strategic investment, such as quality enhancing R&D, marketing and advertisement, which may also improve the future prospects of the firm. Further, our models and results trivially extend to bootstrapped startups. This extension is particularly important because a significant portion of the new firms are financed by bootstrapping (Bhide 2000). We also do not consider venture capital funded startups which may be an interesting future research direction. And, startups in our model do not consider exit strategies, e.g., mergers or acquisition choices, which are also central to process investment decision. It would be useful to explore how investment in process development change without financially risking the startup's survival when the startup's objective is to signal a potentially strong market presence in order to look more attractive for a takeover. This could alter the startup's decisions and yield different results. And finally, it ought to be possible to test the application frameworks in Tables 2 and 3 empirically. Investigating these issues will enhance our understanding of a startup's decision making with regards to product and process R&D management strategies.

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Appendix 1: Proofs

Proof of Proposition 3.1:

Substituting for the first second period monopoly profits, and suppressing the subscripts and the superscripts yields the model:

$$\pi = \max_{q, A \geq 0} (\theta - q - (1+r)c)q - (1+r)A + \left(\frac{\theta - (1+r)(c - \mu A)}{2}\right)^2$$

$$s.t. \quad (\theta - q - (1+r)c)q - (1+r)A \geq 0$$

Now we observe that the single stage problem is separable in q and A , and can be written as:

$$\pi = \max_{q, A \geq 0} f(q) + g(A)$$

$$s.t. \quad A \leq f(q)/(1+r)$$

where $f(q) = (\theta - q - (1+r)c)q$, $g(A) = - (1+r)A + \left(\frac{\theta - (1+r)(c - \mu A)}{2}\right)^2$.

Since $f(q)$ is concave, it is maximized at $q^* = \frac{\theta - (1+r)c}{2}$. Maximizing $f(q)$ also maximizes the right-hand side of the survival constraint, hence q^* gives the optimal quantity decision.

By substituting $q^* = \frac{\theta - (1+r)c}{2}$ into the survival constraint, we obtain the upper bound on the first

period investment as $A_{\max} = \frac{(\theta - (1+r)c)^2}{4(1+r)}$.

Here $g(A)$ is convex and the optimal solution is a boundary solution. Hence the optimal investment amount can be found by evaluating the $g(A)$ function for the values of 0 and A_{\max} and choosing the quantity that maximizes $g(A)$. More explicitly:

$$g(A_{\max}) = -\frac{(\theta - (1+r)c)^2}{4} + \left(\frac{\theta - (1+r)(c - \mu \frac{(\theta - (1+r)c)^2}{4(1+r)})}{2}\right)^2$$

$$g(0) = \frac{(\theta - (1+r)c)^2}{4}$$

$$\Delta^{bc} = g(A_{\max}) - g(0) = \left(\frac{\theta - (1+r)(c - \mu \frac{(\theta - (1+r)c)^2}{4(1+r)})}{2}\right)^2 - \frac{(\theta - (1+r)c)^2}{4}$$

$$= \mu^2(\theta - (1+r)c)^2 + 8\mu(\theta - (1+r)c) - 16$$

So the firm invest A_{\max} if $\Delta^{bc} \geq 0$. Further by factoring Δ^{bc} the optimal threshold policy can be stated as

$$A^* = \begin{cases} A_{\max} & \text{if } \mu(\theta - (1+r)c) \geq 1.66 \\ 0 & \text{o/w} \end{cases}.$$

Finally, substituting the optimal investment policy together with the optimal production quantity to the objective function we obtain:

$$\pi^* = \begin{cases} \left(\frac{\theta - (1+r)(c - \mu A_{\max})}{2} \right)^2 & \text{if } \Delta^{bc} \geq 0 \\ \frac{(\theta - (1+r)c)^2}{2} & \text{o/w} \end{cases}$$

□

Proof of Proposition 3.2:

Following the developments in the proof of Proposition 3.2, when the return on investment is uncertain, first stage problem can be stated as:

$$\begin{aligned} \pi &= \max_{q, A \geq 0} (\theta - q - (1+r)c)q - (1+r)A + \int \frac{(\theta - (1+r)c_2(A_1, \beta))^2}{4} \phi(\beta) d\beta \\ \text{s.t.} \quad & (\theta - q - (1+r)c)q - (1+r)A \geq 0 \end{aligned}$$

Then,

$$\begin{aligned} g(A_{\max}) &= -(1+r)A + \int \frac{(\theta - (1+r)c_2(A_1, \beta))^2}{4} \phi(\beta) d\beta \\ g(0) &= \frac{(\theta - (1+r)c)^2}{4} \\ \Delta^m &= g(A_{\max}) - g(0) = -(1+r)A_{\max} + \int \frac{(\theta - (1+r)c_2(A_{\max}, \beta))^2}{4} \phi(\beta) d\beta - (1+r)A_{\max} \\ &= \int \frac{(\theta - (1+r)(c - \beta A_{\max}))^2}{4} \phi(\beta) d\beta - 2(1+r)A_{\max} \\ &= \int \frac{(\theta - (1+r)c + (1+r)\beta A_{\max})^2}{4} \phi(\beta) d\beta - 2(1+r)A_{\max} \\ &= \int \frac{(\theta - (1+r)c)^2 + (1+r)^2 \beta^2 A_{\max}^2 + 2(\theta - (1+r)c)(1+r)\beta A_{\max}}{4} \phi(\beta) d\beta - 2(1+r)A_{\max} \\ &= \frac{(\theta - (1+r)c)^2}{4} + \frac{(1+r)^2(\sigma^2 + \mu^2)A_{\max}^2 + 2(\theta - (1+r)c)(1+r)\mu A_{\max}}{4} - 2(1+r)A_{\max} \\ &= \frac{(1+r)^2(\sigma^2 + \mu^2)A_{\max}^2 + 2(\theta - (1+r)c)(1+r)\mu A_{\max}}{4} - (1+r)A_{\max} \\ &= (\sigma^2 + \mu^2) \frac{(\theta - (1+r)c)^2}{4} + 2(\theta - (1+r)c)\mu - 4 = (\sigma^2 + \mu^2)(\theta - (1+r)c)^2 + 8(\theta - (1+r)c)\mu - 16 \\ &= \Delta^{bc} + \sigma^2(\theta - (1+r)c)^2 \end{aligned}$$

□

Proof of Proposition 3.3:

i) Since a Cournot game is played in the second period, startup's and competitor's equilibrium quantities are given by $q_2 = \frac{\theta + \xi - 2(1+r)c_2(A_1, \beta)}{3}$ and $q_c = \frac{\theta + (1+r)c_2(A_1, \beta) - 2\xi}{3}$, respectively.

Consequently, startup's profit in the second period is :

$$\pi_2(A_1, \xi, \beta) = \frac{(\theta + \xi - 2(1+r)c_2(A_1, \beta))^2}{9}$$

Hence, substituting the expected second period profit, the first stage problem becomes:

$$\begin{aligned} \pi &= \max_{q, A \geq 0} (\theta - q - (1+r)c)q - (1+r)A + \int \frac{(\theta + \xi - 2(1+r)c_2(A, \beta))^2}{9} \phi(\xi) d\xi \\ \text{s.t.} \quad & (\theta - q - (1+r)c)q - (1+r)A \geq 0 \end{aligned}$$

Then, following the developments in the proof of Proposition 3.2 above, it is still optimal to produce the monopoly level and, the investment threshold and optimal profits are obtained as follows:

$$\begin{aligned} g(A_{\max}) &= -(1+r)A_{\max} + \int \frac{(\theta + \xi - 2(1+r)(c - \mu A_{\max}))^2}{9} \phi(\xi) d\xi \\ g(0) &= \int \frac{(\theta + \xi - 2(1+r)c)^2}{9} \phi(\xi) d\xi \\ \Delta^c &= g(A_{\max}) - g(0) = -(1+r)A_{\max} + \int \frac{(\theta + \xi - 2(1+r)(c - \mu A_{\max}))^2}{9} \phi(\xi) d\xi - \int \frac{(\theta + \xi - 2(1+r)c)^2}{9} \phi(\xi) d\xi \\ &= -(1+r)A_{\max} + \int \left\{ \frac{(\theta + \xi - 2(1+r)c + 2(1+r)\mu A_{\max})^2 - (\theta + \xi - 2(1+r)c)^2}{9} \right\} \phi(\xi) d\xi \\ &= -(1+r)A_{\max} + \frac{2(\theta + \lambda - 2(1+r)c)2(1+r)\mu A_{\max} + 4(1+r)^2 \mu^2 (A_{\max})^2}{9} \\ &= -9 + 2(\theta + \lambda - 2(1+r)c)2\mu + 4(1+r)\mu^2 A_{\max} \\ &= -9 + 4\mu(\theta + \lambda - 2(1+r)c) + \mu^2(\theta - (1+r)c)^2 \\ &= (\mu(\theta - (1+r)c) + 2)^2 - 4\mu((1+r)c - \lambda) - 13 \end{aligned}$$

So the firm invest A_{\max} if $\Delta^c \geq 0$. Finally, substituting the optimal investment policy together with the optimal production quantity to the objective function we obtain:

$$\pi^* = \begin{cases} \frac{1}{9} \{ (\theta - 2(1+r)c + \lambda)^2 + 4A_{\max} \{ \mu(\theta - 2(1+r)c + \lambda) + \mu^2 A_{\max} \} + \tau^2 \} & \text{if } \Delta^c \geq 0 \\ \frac{(\theta - (1+r)c)^2}{4} + \frac{1}{9} \{ (\theta - 2(1+r)c + \lambda)^2 + \tau^2 \} & \text{o/w} \end{cases}$$

□

Proof of Corollary 3.4:

Note that Δ^c is linear increasing in λ . Hence solving $\Delta^{bc} - \Delta^c = 0$ for λ , we find that

$$\Delta^{bc} \geq \Delta^c \quad \text{if} \quad \lambda \leq \theta - 7/4\mu \text{ and,}$$

$$\Delta^{bc} \leq \Delta^c \quad \text{if} \quad \lambda \geq \theta - 7/4\mu$$

(We assume parameters guarantee positive quantities and positive second period cost, i.e., q_2 , q_c and c_2 are non-negative.) \square

Proof of Proposition 4.1:

Below, we evaluate the expectation step by step and simplify the problem (4):

$$\max_{q_1, A_1} E_{\tilde{\varepsilon}_1} \{ (p(q_1, \tilde{\varepsilon}_1) - c_1)q_1 - r(c_1 q_1 + A_1) - A_1 \} + \alpha E_{\tilde{\xi}, \tilde{\beta}, \tilde{\varepsilon}_1} \left\{ \frac{(\theta + \tilde{\xi} - 2(1+r)c_2(A_1, \tilde{\beta}))^2}{9} \mid \pi_1(\tilde{\varepsilon}_1) \geq \underline{\pi} \right\}$$

$$s.t. \quad q_1, A_1 \geq 0$$

$$\max_{q_1, A_1} E_{\tilde{\varepsilon}_1} \{ (p(q_1, \tilde{\varepsilon}_1) - c_1)q_1 - r(c_1 q_1 + A_1) - A_1 \} + \alpha \int_B \int_0^\infty \int_0^\infty \frac{((\theta + \xi - 2(1+r)c_2(A_1, \beta))^2}{9} \phi(\xi) \psi(\beta) \varphi(\varepsilon_1) d\xi d\beta d\varepsilon_1$$

$$s.t. \quad q_1, A_1 \geq 0$$

where $B = \{ \varepsilon_1 \mid \pi_1(\varepsilon_1) \geq \underline{\pi} \}$

$$\max_{q_1, A_1} E_{\tilde{\varepsilon}_1} \{ (p(q_1, \tilde{\varepsilon}_1) - c_1)q_1 - r(c_1 q_1 + A_1) - A_1 \} + \alpha \int_B \int_0^\infty \frac{\tau^2 + (\theta + \lambda - kc_2(A_1, \beta))^2}{9} \psi(\beta) \varphi(\varepsilon_1) d\beta d\varepsilon_1$$

$$s.t. \quad q_1, A_1 \geq 0$$

$$\max_{q_1, A_1} E_{\tilde{\varepsilon}_1} \{ (p(q_1, \tilde{\varepsilon}_1) - c_1)q_1 - r(c_1 q_1 + A_1) - A_1 \} +$$

$$\alpha \int_B \frac{\tau^2 + (\theta + \lambda - kc_1)^2 + 2(\theta + \lambda - kc_1)k\mu A_1 + k^2(\mu^2 + \sigma^2)A_1^2}{9} \varphi(\varepsilon_1) d\varepsilon_1$$

$$s.t. \quad q_1 \geq 0, A_1 \geq 0$$

$$\max_{q_1, A_1} E_{\tilde{\varepsilon}_1} \{ (p(q_1, \tilde{\varepsilon}_1) - c_1)q_1 - r(c_1 q_1 + A_1) - A_1 \} +$$

$$\alpha \left[\frac{\tau^2 + (\theta + \lambda - kc_1)^2 + 2(\theta + \lambda - kc_1)k\mu A_1 + k^2(\mu^2 + \sigma^2)A_1^2}{9} \right] (1 - \mathcal{G}_{\varepsilon_1} \left(\frac{\underline{\pi} + (1+r)(c_1 q_1 + A_1)}{q_1} + q_1 - \theta \right))$$

$$s.t. \quad q_1 \geq 0, A_1 \geq 0$$

$$\max_{q_1, A_1} [(\theta - q_1)q_1 - (r+1)(c_1 q_1 + A_1)] +$$

$$\alpha \left[\frac{\tau^2 + (\theta + \lambda - kc_1)^2 + 2(\theta + \lambda - kc_1)k\mu A_1 + k^2(\mu^2 + \sigma^2)A_1^2}{9} \right] (1 - \mathcal{G}_{\varepsilon_1} \left(\frac{\underline{\pi} + (1+r)(c_1 q_1 + A_1)}{q_1} + q_1 - \theta \right))$$

$$s.t. \quad q_1 \geq 0, A_1 \geq 0$$

Finally, suppressing the subscripts and superscripts, we obtain the result:

$$\max_{q, A} f(q, A) = [(\theta - q)q - (r+1)(cq + A)] +$$

$$\alpha \left[\frac{\tau^2 + (\theta + \lambda - kc)^2 + 2(\theta + \lambda - kc)k\mu A + k^2(\mu^2 + \sigma^2)A^2}{9} \right] (1 - \mathcal{G}_{\varepsilon_1} \left(\frac{\underline{\pi} + (1+r)(cq + A)}{q} + q - \theta \right))$$

$$s.t. \quad q \geq 0, A \geq 0$$

Recall that $\pi_m = E[(p(q_m, \tilde{\varepsilon}_1) - (1+r)c)q_m]$, $q_m = \frac{(\theta - (1+r)c)}{2}$, $A_m = \frac{\pi_m - \underline{\pi}}{(1+r)}$. First, we partition the

feasible decision space into the following four regions:

- (i) $q \leq q_m$ and $A \leq A_m$
- (ii) $q \leq q_m$ and $A \geq A_m$
- (iii) $q \geq q_m$ and $A \leq A_m$
- (iv) $q \geq q_m$ and $A \geq A_m$

Following the developments of the optimization problem in Proposition 4.1, let

$$D = \alpha \left[\frac{\tau^2 + (\theta + \lambda - kc)^2 + 2(\theta + \lambda - kc)k\mu A + k^2(\mu^2 + \sigma^2)A^2}{9} \right].$$

Then, at the optimality it must hold that:

$$\frac{\partial f(q, A)}{\partial q} = \theta - 2q - (r+1)c - D\varphi\left(\frac{\pi + (1+r)(cq + A)}{q} + q - \theta\right) * \left(1 - \frac{\pi + (1+r)A}{q^2}\right) = 0.$$

It is easy to observe that $\frac{\partial f(q, A)}{\partial q} > 0$ for every decision vector in region (ii) and $\frac{\partial f(q, A)}{\partial q} < 0$ for every decision vector in region (iii). Hence the optimal policy should either lie in region (i) or (iv). This concludes the desired “if and only if” argument in the proposition.

Note that the proof does not depend on the convexity of $f(q, A)$. Indeed, $f(q, A)$ is not generally convex and in this proof, we don't address the optimal investment, A^* . Recall that the joint optimization of A and q is not tractable analytically. Indeed, numeric analyses show that at the optimality, when both A and q varied simultaneously, optimal policy may be either in region (i) and (iv). \square

Proof of Proposition 4.2:

Recall that the demand shock $\tilde{\varepsilon}_t$ ($t = 1, 2$) has a normal probability density function, $\varphi(\cdot)$, with mean zero and variance v^2 . It is sufficient to show that the argument of the survival probability,

$$\frac{\pi + (1+r)A}{q} + q - \theta + (1+r)c, \text{ is less (greater) than or equal to zero if and only if the optimal operating}$$

policy is aggressive (conservative).

Let $x = \frac{\pi + (1+r)A}{q} + q - \theta + (1+r)c$. Since $q_m = \frac{\theta - (1+r)c}{2}$, we can state the argument of the survival

probability as $x = \frac{\pi + (1+r)A}{q} + q - 2q_m$. First observe that the first order optimality condition

$$\frac{\partial f(q, A)}{\partial q} = \theta - 2q - (r+1)c - D\varphi\left(\frac{\pi + (1+r)(cq + A)}{q} + q - \theta\right) * \left(1 - \frac{\pi + (1+r)A}{q^2}\right) = 0 \quad \text{implies that}$$

$$A \geq \frac{q^2 - \pi}{1+r} \text{ if } q \geq q_m, \text{ and } A \leq \frac{q^2 - \pi}{1+r} \text{ if } q \leq q_m.$$

Now, suppose the optimal operating policy is aggressive. By definition $q \geq q_m$ and $A \geq \frac{q^2 - \underline{\pi}}{1+r} \geq A_m$.

Then, if the optimal policy is aggressive, the first two terms of x is larger than $2q_m$ and hence, $\frac{\underline{\pi} + (1+r)A}{q} + q - \theta + (1+r)c > 0$.

Suppose the optimal operating policy is conservative. By definition $q < q_m$ and $A \leq \frac{q^2 - \underline{\pi}}{1+r} \leq A_m$. Then,

the first two terms of x is less than $2q_m$ and hence, $\frac{\underline{\pi} + (1+r)A}{q} + q - \theta + (1+r)c < 0$.

Now we prove the *only if* part of the argument.

Consider an optimal operating policy (A, q) such that $x = \frac{\underline{\pi} + (1+r)A}{q} + q - 2q_m > 0$, i.e., the survival

chances are less than 50 percent. Now, suppose this optimal operating policy is conservative, i.e., $q \leq q_m$

and $A \leq \frac{q^2 - \underline{\pi}}{1+r} \leq A_m$. Then, $\frac{\underline{\pi} + (1+r)A}{q} < q$ and hence, $\frac{\underline{\pi} + (1+r)A}{q} + q - 2q_m < 0$. So, by

contradiction, the policy should be aggressive.

The conservative case can be proven similarly. \square

Proofs of Corollary 4.1 and Corollary 4.2:

Immediate economical viability, $\pi_m - \underline{\pi} > 0$, implies that $A_m = \frac{\pi_m - \underline{\pi}}{(1+r)} > 0$. When there are no

investment opportunities firm has to set $A = 0$ and hence $A < A_m$. From Proposition 4.1, this implies that the optimal policy should be in region (i) and $q < q_m$. When there are investment opportunities, i.e., $A \geq 0$, the optimal solution may be in either region (i) or (iv). Indeed, in Section 5, we provide various numerical examples for both cases.

On the other hand, if $\pi_m - \underline{\pi} < 0$, then $A_m = \frac{\pi_m - \underline{\pi}}{(1+r)} < 0$. In this case, regardless of the existence of

investment opportunities $A > A_m$ and hence, the optimal policy should lie in region (iv) and $q > q_m$. \square

Proof of Proposition 5.1:

i) Let $f(q) = (\theta - q - c)q$, and $g(A) = -A + \left(\frac{\theta - (c - \mu A)}{2}\right)^2$. Then the maximization problem can be

stated as:

$$\max_{q,A} f(q) + g(A)$$

$$s.t. \quad A \leq L - cq \quad (1)$$

$$A \leq f(q) \quad (2)$$

$$A, q \geq 0$$

$$\max f(q) + g(A)$$

$$\text{Or equivalently, } s.t. \quad A \leq \min \{f(q), L - cq\}$$

$$A, q \geq 0$$

Solving $f(q) = L - cq$ for q , we obtain $q_1 = \frac{\theta}{2} - \frac{\sqrt{\theta^2 - 4L}}{2}$ and $q_2 = \frac{\theta}{2} + \frac{\sqrt{\theta^2 - 4L}}{2} \geq q_m$. Then, if $q_1 \geq q_m$, the debt constraint never binds as shown in Figure A1. Hence Proposition 3.1 applies. More specifically, this case can be stated as:

$$\frac{\theta}{2} - \frac{\sqrt{\theta^2 - 4L}}{2} \geq \frac{\theta - c}{2} \rightarrow L \geq \frac{\theta^2 - c^2}{4} = cq_m + A_{\max}$$

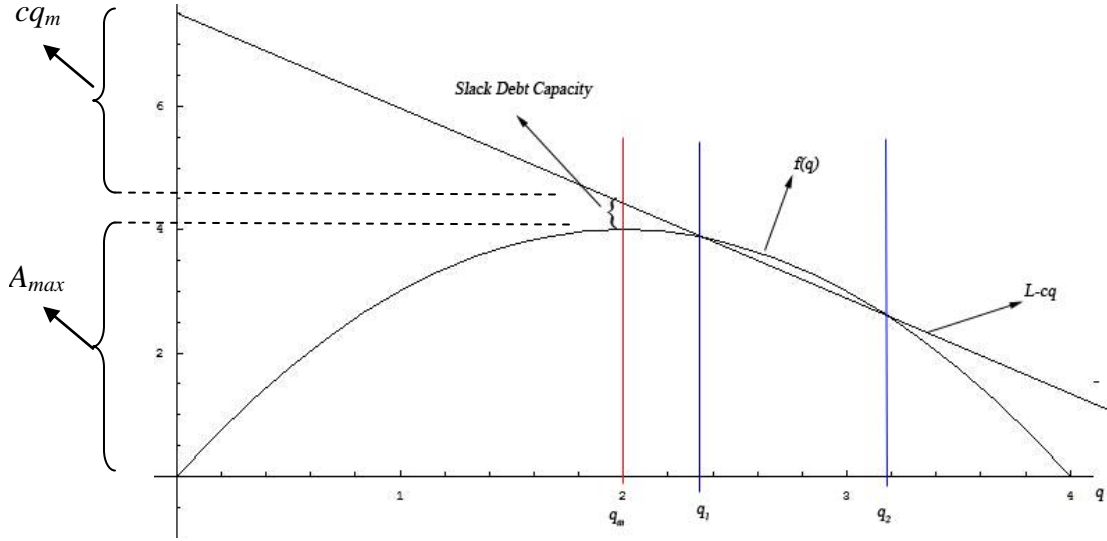


Figure A1

ii) If $L < cq_m$, then the debt constraint should always bind since any excess cash can be used to increase production and increase profits.

iii) Suppose $cq_m \leq L \leq cq_m + A_{\max}$ and the return on investment is very close to zero. Then, the firm simply produces up to the monopoly level and does not use the rest of the debt capacity for investment. Hence, the debt capacity constraint may not be tight. \square

Proof of Corollary 5.2:

From the proof of Proposition 3.1, when there is no debt capacity, investment threshold is given by: $\Delta = g(A_{\max}) - g(0)$. When there is an explicit debt capacity, L , then for any feasible of production policy q , maximum investment budget is given by: $A_{\max}(q, L) = \min\{f(q), L - cq\} < A_{\max}$. Next, since $g(\cdot)$ is increasing in A , $\Delta^l(q, L) = g(A_{\max}(q, L)) - g(0) \leq \Delta$. \square

Appendix 2: Computational Procedure

We work with (4):

$$z^* = \max_{q_1, A_1} E_{\tilde{\varepsilon}_1} \{ (p(q_1, \tilde{\varepsilon}_1) - c_1)q_1 - r(c_1 q_1 + A_1) - A_1 \} + \alpha E_{\tilde{\xi}, \tilde{\beta}, \tilde{\varepsilon}_1} \left\{ \frac{(\theta + \tilde{\xi} - 2(1+r)c_2(A_1, \tilde{\beta}; A_0))^2}{9} \mid \pi_1(\tilde{\varepsilon}_1) \geq \underline{\pi} \right\}$$

s.t. $q_1, A_1 \geq 0$

Since this optimization problem has only two decision variables, we implement a search algorithm, in C++, to find the optimal decisions for each set of parameter values. This algorithm is available from the authors on request. For example, in order to construct Figure 6, we first fix μ and then solve (4) optimally for varying values of λ , and we determine the critical value(s) of λ after which the firm switches its optimal investment strategy. Then, by connecting these critical values with a smooth line we arrive at Figure 6.